CHAPTER 1
CLASSIFICATION OF INDUSTRIES AND PRODUCTS

A model of industrial organization analysis: ysis:
(FS Cht)

Structuralist:
1. The inclusion of conduct variables is not essential to the development of an operational theory of industrial organization.
2. a priori theory based upon structure-conduct and conduct-performance links yields ambiguous predictions.
3. Even if a priori structure-conduct-performance hypotheses could be formulated, attempting to test these hypotheses would encounter serious obstacles.

Behaviorist: We can do still better with a richer model that includes intermediate behavioral links.

Law and Economics
Antitrust law,
Patent and Intellectual Property protection
Cyber law or Internet Law

Industrial Organization and International Trade

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CHAPTER 2
TWO SIDES OF A MARKET

2 Two Sides of a Market
2.1 Comparative Static Analysis
Assume that there are n endogenous variables and m exogenous variables.
Endogenous variables: \(x_1, x_2, \ldots, x_n\)
Exogenous variables: \(y_1, y_2, \ldots, y_m\)

There should be equations so that the model can be solved:
\[
P_1(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) = 0
\]
\[
P_2(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) = 0
\]
\[
P_n(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) = 0.
\]

Some of the equations are behavioral, some are equilibrium conditions, and some are definitions.

In principle, given the values of the exogenous variables, we solve to find the endogenous variables as functions of the exogenous variables:
\[
x_1 = x_1(y_1, y_2, \ldots, y_m)
\]
\[
x_2 = x_2(y_1, y_2, \ldots, y_m)
\]
\[
\vdots
\]
\[
x_n = x_n(y_1, y_2, \ldots, y_m).
\]

We use comparative statics method to find the differential relationships between \(x_i\) and \(y_j\): \(\partial x_i / \partial y_j\). Then we check the sign of \(\partial x_i / \partial y_j\) to investigate the causality relationship between \(x_i\) and \(y_j\).

2.2 Utility Maximization and Demand Function

2.2.1 Single product case
A consumer wants to maximize his/her utility function \(U = u(Q) + M = u(Q) + (Y - PQ)\).

FOC: \(\frac{\partial U}{\partial Q} = u'(Q) = P\) (inverse demand function)

\(\Rightarrow M = D(P)[\text{demand function, a behavioral equation}]
\)

\(\frac{\partial^2 U}{\partial Q^2} = -1 \Rightarrow \frac{\partial^2 U}{\partial Q^2} = D'(P) < 0\), the demand function is a decreasing function of price.

2.2.2 Multi-product case
A consumer wants to maximize his utility function subject to his budget constraint:
\[
\max U(x_1, \ldots, x_n) \quad \text{subject to } p_1 x_1 + \cdots + p_n x_n = I.
\]

Endogenous variables: \(x_1, x_2, \ldots, x_n\)
Exogenous variables: \(p_1, \ldots, p_n, I\) (the consumer is a price taker)

Solution is the demand functions \(x_k = D_k(p_1, \ldots, p_n, I)\), \(k = 1, \ldots, n\)

Example: max \(U(x_1, x_2) = a \ln x_1 + b \ln x_2\) subject to \(p_1 x_1 + p_2 x_2 = M\).

\(L = a \ln x_1 + b \ln x_2 + \lambda (m - p_1 x_1 - p_2 x_2)\).

\(\Rightarrow \frac{\partial L}{\partial x_1} = \frac{a}{x_1} = \frac{\partial L}{\partial p_1} = 0\), \(\Rightarrow \frac{\partial L}{\partial x_2} = \frac{b}{x_2} - \lambda = 0\) and \(\Rightarrow m = -p_1 x_1 - p_2 x_2 = 0\).

\(\Rightarrow \frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial p_1} = \frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial p_2} = 0\).

\(\Rightarrow x_1 = \frac{m}{p_1}, x_2 = \frac{m}{p_2}\).
2.3 Indivisibility, Reservation Price, and Demand Function
In many applications the product is indivisible and every consumer needs at most one unit.
Reservation price, the value of one unit to a consumer.
If we rank consumers according to their reservation prices, we can derive the market demand function.
Example: $l_i = 31 - i, i = 1, 2, \ldots, 30$

2.4 Demand Function and Consumer surplus
Demand Function: $Q = D(p)$.
Inverse demand function: $p = P(Q)$.
Demand elasticity: $ep = \frac{\partial Q}{\partial p} \frac{P}{Q}$.
Total Revenue: $TR(Q) = PQ, MP = \frac{\partial TR}{\partial Q} = P$.
Average Revenue: $AR(Q) = \frac{TR(Q)}{Q}, MR = \frac{\partial TR}{\partial Q} = \frac{P}{Q}$.
Marginal Revenue: $MR(Q) = \frac{\partial TR}{\partial Q}$ or $MR(Q) = P(Q) + Q\frac{\partial P}{\partial Q}$.

2.5 Profit maximization and supply function
2.5.1 From cost function to supply function
Consider first the profit maximization problem of a competitive producer:
\[ \max \Pi = PQ - C(Q). \]
\[ \text{FOC: } \frac{\partial \Pi}{\partial Q} = P - \frac{\partial C}{\partial Q} = 0. \]
The FOC is the inverse supply function (a behavioral equation) of the producer: $P = C'(Q) = MC$. Remember that $Q$ is endogenous and $P$ is exogenous here. To find the comparative statics $\frac{\partial Q}{\partial P}$, we use the total differential method discussed in the last chapter:
\[ dP = C'(Q)dQ, \quad \frac{\partial Q}{\partial P} = \frac{1}{C'(Q)}. \]
To determine the sign of $\frac{\partial Q}{\partial P}$, we need the SOC, which is $\frac{\partial P}{\partial Q} = -C''(Q) < 0$.

2.5.2 From production function to cost function
A producer's production technology can be represented by a production function
$q = f(x_1, \ldots, x_k)$. Given the prices, the producer maximizes his profits:
\[ \max \Pi = PQ - C(Q), \text{ where } P = f(x_1, \ldots, x_k). \]
Exogenous variables: $p_1, p_2, \ldots, p_n$ (the producer is a price taker).
Solution: The supply is the function $q = S(p_1, p_2, \ldots, p_n)$ and the input demand functions:
$x_k = X_k(p_1, p_2, \ldots, p_n), k = 1, \ldots, n$.
Example: $q = f(x_1, x_2) = 2\sqrt{x_1} + 2\sqrt{x_2}$ and $P(x_1, x_2, p_1, p_2, p_3) = \frac{p_1}{2\sqrt{x_1}} + \frac{p_2}{2\sqrt{x_2}} + p_3$.
\[ \text{FOC: } \frac{\partial P}{\partial x_2} = \frac{p_1}{2\sqrt{x_1}} - \frac{p_2}{2\sqrt{x_2}} = \text{subject to } \frac{\partial P}{\partial x_2} = 0. \]
To solve the maximization problem, we can eliminate $x_2$: $x_2 = \frac{p_1}{p_2} x_1$ and $x_2 = \frac{p_3}{p_1} x_1$.
Supporting factors: $\frac{\partial Q}{\partial P} > 0$. Substituting factors $\frac{\partial Q}{\partial P} < 0$.
Examples: 1. Cobb-Douglas case $Q(L, K) = AL^\alpha K^\beta$.
Example 2: CES case $Q(L, K) = (\alpha M_L + (1 - \alpha) M_K)^{1/(1-\alpha)}$.

2.7 Cost function: $C(Q)$
Total cost: $TC = C(Q)$ Average cost: $AC = \frac{C(Q)}{Q}$ Marginal cost: $MC = C'(Q)$.
Examples: 1. $C(Q) = F + eQ$.
Example 2: $C(Q) = F + eQ + bQ^2$.
Example 3: $C(Q) = eQ^2$.

CHAPTER 3

COMPETITIVE MARKET

3 Competitive Market
Industry (Market) structure:
Short Run: Number of firms, distribution of market share, competition decision variables, reactions to other firms.
Long Run: R&D, entry and exit barriers.
Competition: In the SR, firms and consumers are price takers.
In the LR, there is no barriers to entry and exit $\rightarrow$ L-imperfect.
3.1 SR market equilibrium

3.1.1 An individual firm’s supply function

A producer \( i \) in a competitive market is a price taker. It chooses its quantity to maximize its profit:

\[
\max_{Q_i} p Q_i - C_i(Q_i) \Rightarrow p = C_i(Q_i) \Rightarrow Q_i = S_i(p).
\]

3.1.2 Market supply function

Market supply is the sum of individual supply function \( S(p) = \sum_i S_i(p) \).

On the \( Q-p \) diagram, it is the horizontal sum of individual supply curves.

3.2 Market equilibrium

Market equilibrium is determined by the intersection of the supply and demand as in the diagram.

Formally, suppose there are \( n \) firms. A state of the market is a vector \((p, Q_1, Q_2, \ldots, Q_n)\).

An equilibrium is a state \((p^*, Q_1^*, Q_2^*, \ldots, Q_n^*)\) such that:

1. \( D(p^*) = \sum_i S_i(p^*) \).
2. Each firm's profit maximization problem:
   \[
   \max_{Q_i} p_i Q_i - C_i(Q_i) = \sum_{i=1}^{n} \max_{Q_i} p_i Q_i - C_i(Q_i) \geq 0.
   \]
3. Example 1: \( Q_i(Q) = Q_i, C_i(Q_i) = 2Q_i, D_i = \frac{12 - Q_i}{4} \)

\[
p = C_i(Q_i) = 2Q_i, p = C_i(Q_i) = 2Q_i \Rightarrow S_i = \frac{p}{2}, S_i = \frac{Q_i}{2}, S_i(p) = S_i + \frac{3p}{4}
\]

\[
D_i(p^*) - S_i(p^*) = 12 - \frac{3p^*}{4} = p^* - 12, Q_i = S_i(p^*) = 0, Q_i = S_i(p^*) = 0, Q_i = S_i(p^*) = 0
\]

3.2.2 Example 2: \( U_i(Q) = Q_i (COTS) \) and \( D_i(p) = \max[0 - A - bQ_i] \).

If production technology is COTS, then the equilibrium market price is determined by the AC and the equilibrium quantity is determined by market demand.

\[
p^* = c, q^* = D_i(p^*) = \max[0 - A - bQ_i] > 0.
\]

If \( \frac{A}{b} < c \) then \( q^* = 0. \) If \( \frac{A}{b} > c \) then \( q^* = \frac{A}{b} - a > 0. \)

3.2.3 Example 3: 2 firms, \( C_i(Q_i) = a, C_2(Q_2) = cQ_2, c < a \)

\[
p^* = c_1, Q^* = \frac{3Q_1}{4} = D_i(p^*) = D_i(c_i), q^* = 0.
\]

3.2.4 Example 4: \( C_i(Q) = F + aQ_i \) or \( C_i(Q) > 0 \) (HITS), no equilibrium

If \( C_i(Q) > 0 \), then the profit maximization problem has no solution.

\[
\text{If } C_i(Q) = F + aQ_i, \text{ then } p^* = c \text{ cannot be an equilibrium because } \Pi_i(Q_i) = C_i(Q_i) - (F + aQ_i) = -F. \]

3.3 General competitive equilibrium

Commodity space. Assume that there are \( n \) commodities. The commodity space \( R_n = \{(x_1, \ldots, x_n); x_i \geq 0 \} \).

Economy: There are \( I \) consumers, \( J \) producers, with initial endowments of commodities \( w = (w_1, \ldots, w_J) \).

Consumer \( i \) has a utility function \( U_i(x_1, \ldots, x_J) \).

Producer \( j \) has a production transformation function \( A_j(x_1, \ldots, x_J) = 0 \).

A price system: \((p_1, \ldots, p_n)\).

A private ownership economy: Endowments and firms (producers) are owned by consumers.

Consumer i’s endowment is \( w_i = \{w_{i1}, \ldots, w_{iJ}\} \).

Consumer i’s share of firm j’s output is \( \theta_i = \frac{w_{ij}}{\sum_j w_{ij}} \).

An allocation: \( x^* = \{x_{i1}, \ldots, x_{iJ}\} \).

A competitive equilibrium:

A combination of a price system \( p = (p_1, \ldots, p_n) \) and an allocation \( (x^1, \ldots, x^J) \) such that:

1. \( \sum_j x_{ij} = w_{ij} \) (feasibility condition).
2. \( x^*_j \) maximizes \( I_j \), \( j = 1, \ldots, J \) and \( x^*_j \) maximizes \( I_j \), subject to \( x^*_j \) budget constraint \( \sum_j p_j x_{ij} = \sum_j p_j x_j + \sum_j \theta_i p_j x_j = \sum_j \theta_i p_j x_j \).

Existence Theorem:

Suppose that the utility functions are quasiconcave and the production transformation functions satisfy some other analytic conditions, then a competitive equilibrium exists.

Welfare Theorem: A competitive equilibrium is efficient and an efficient allocation can be achieved as a competitive equilibrium through the production-consumption process.

Constant returns to scale economies and non-substitution theorems:

Suppose there is only one nonproduced input, this input is indispensable to production, there is no joint production, and the production functions exhibit constant returns to scale. Then the competitive equilibrium price system is determined by the production side only.

CHAPTER 4

MONOPOLY

4 Monopoly

A monopoly industry consists of one single producer who is a price setter (in order to gain monopoly power to control market price).

4.1 Monopoly profit maximization

Let the market demand of a monopoly be \( Q = D(p) \) with inverse function \( P = f(Q) \).

Its total cost is \( TC = C(Q) \). The profit maximization problem is

\[
\max_{Q} \pi(Q) = PQ - TC = f(Q)Q - C(Q) = f(Q)Q + f'(Q) - MR(Q) = MC(Q) - C(Q) = Q_0.
\]

The SOC is \( \frac{d\pi}{dQ} = MR(Q) = MC(Q) < 0 \).

Long-run equilibrium condition: \( \pi(Q_0) \geq 0 \).

Example: \( TC(Q) = F + aQ + bQ^2, f(Q) = a - bQ \Rightarrow MC = 2Q, MR = a - 2bQ \).

\[
Q_0 = \frac{a}{2(b+c)} \Rightarrow f(Q_0) = a - \frac{a^2}{2(b+c)} \Rightarrow f'(Q_0) = \frac{a}{b+c} \Rightarrow f(Q_0) = f'(Q_0) - F.
\]

When \( \frac{a}{b+c} < F \), the true solution is \( Q_0 = 0 \) and the market does not exist.

4.1.1 Lerner index

The maximization can be solved using \( P \) as independent variable:

\[
\max_{P} \pi(P) = PQ - TC = PD(P) - C(D(P)) = D(P) + PD(P) - C(D(P)) \Rightarrow \frac{\partial \pi(P)}{\partial P} = \frac{D(P)}{D(P)} - 1 \Rightarrow L = \frac{D(P)}{D(P)} - 1
\]

Lerner index: \( \frac{P - C}{P} = L \). It can be calculated from real data for a firm (not necessarily monopoly) or an industry. It measures the profit per dollar sale of a firm (or an industry).
4.1.2 Monopoly and social welfare

4.1.3 Rent seeking (を利用する) activities

R&D, Bribery, Persuasive advertising, Excess capacity to discourage entry, Lobby expenses. Over using R&D, etc are means taken by firms to secure and/or maintain their monopoly profits. They are called rent seeking activities because monopoly profit is similar to land rent. They are in many cases regarded as wastes because they don’t contribute to improving productivities.

4.2 Monopoly price discrimination

Indiscriminate Pricing: The same price is charged for every unit of a product sold to any consumer.

Third degree price discrimination: Different prices are set for different consumers, but the same price is charged for every unit sold to the same consumer (linear pricing). Second degree price discrimination: Different price is charged for different units sold to the same consumer (non-linear pricing). But the same price schedule is set for different consumers.

First degree price discrimination: Different price is charged for different units sold to the same consumer (non-linear pricing). In addition, different price schedules are set for different consumers.

4.2.1 Third degree price discrimination

Assume that a monopoly sells its product in two separable markets.

Cost function: \( C(Q) = (q + q^2) \)

Inverse market demand: \( P_1 = D_1(q_1) = -q_1 + 2q_1^2 \) and \( P_2 = D_2(q_2) \)

Profit function: \( \Pi(q_1, q_2) = \Pi_1(q_1) + \Pi_2(q_2) - C(q_1 + q_2) \)

FOC: \( \Pi_1 = \frac{d}{dq_1}[\Pi(q_1, q_2)] = -1 + 4q_1 = 0 \), \( \Pi_2 = \frac{d}{dq_2}[\Pi(q_1, q_2)] = -1 + 4q_2 = 0 \), or \( MR_1 = MR_2 = MC_1 = MC_2 \).

SOC: \( MR_1 = P_1 = C'_{q_1} = \frac{d}{dq_1}[C(q_1 + q_2)] = \frac{d}{dq_1}[q_1 + q_2^2] = 1 + 2q_1 \)

Example: \( q_1 = \alpha - 2q_1 + q_2 \), \( q_2 = \beta - q_2, C(Q) = 0.5Q^2 + 0.5C(q_1 + q_2) \), \( \frac{d}{dq_1}[C(q_1 + q_2)] = \frac{d}{dq_1}[0.5q_1^2 + 0.5q_2^2] = q_1 + q_2 \), and \( C'_{q_1} = 1 + 2q_1 = 0 \).

4.2.2 First Degree

Each consumer is charged according to his total utility, i.e., \( TR = PQ = U(Q) \). The total profit to the monopoly is \( \Pi(Q) = U(Q) - C(Q) \). The FOC is \( U'(Q) = C'(Q) \), i.e., the monopoly regards a consumer’s MU \( U'(Q) \) curve as its MR curve and maximizes its profit.

\[ \max \Pi = U(Q) - C(Q) = U'(Q) \]

The profit maximizing quantity is the same as the competition case, \( Q_{eq} = Q^* \). However, the price is much higher, \( \frac{P_{eq}}{U'(Q^*)} = P^* = U'(Q^*) \). There is no inefficiency. But there is social justice problem.

4.2.3 Second degree discrimination

By self selection principle, \( P_1Q_1 = A, P_2Q_2 = A + C, \Pi = 2A + C \) is maximized when \( Q_1 \) is such that the height of \( D_2 \) is twice that of \( D_1 \).

4.3 Multiplant Monopoly and Cartel

Now consider the case in which a monopoly has two plants.

Cost functions: \( TC_1 = C_1(q_1) \) and \( TC_2 = C_2(q_2) \)

Inverse market demand: \( P_1 = D_1(q_1) = -q_1 + 2q_1^2 \) and \( P_2 = D_2(q_2) \)

Profit function: \( \Pi(q_1, q_2) = P_1(D_1(q_1)) + D_2(q_2) - C_1(q_1) - C_2(q_2) \)

FOC: \( \Pi_1 = -D_1(q_1) - C_1'(q_1) = 0, \Pi_2 = -D_2(q_2) - C_2'(q_2) = 0, \)

SOC: \( MR_1 = 3D_1'(q_1) + 2D_2'(q_2) = C_1'(q_1) = C_2'(q_2) \)

Example: \( D_1(q_1) = -q_1 + 2q_1^2, C_1(q_1) = aq_1, C_2(q_2) = 2q_2^2 \)

FOC: \( MR = -2q_1 + 2q_1^2 + 2q_2^2 = 2q_1 = 4q_2 = 2q_1 + 2q_2^2 = 4q_1 \)

When \( q_1 = 0.2q_1, q_2 = 0.1q_2, P_{eq} = 0.7A \)

4.4 Multiproduct monopoly

Consider a producer who is monopoly (the only seller) in two joint products, \( Q_1 = D_1(P_1, P_2), Q_2 = D_2(P_1, P_2) \), \( TC = C(Q_1, Q_2) \).

The profit as a function of \( (P_1, P_2) \) is

\[ \Pi(P_1, P_2) = P_1D_1(P_1, P_2) + P_2D_2(P_1, P_2) - C(D_1(P_1, P_2), D_2(P_1, P_2)) \]

Maximizing \( \Pi(P_1, P_2) \) w. r. t. we have

\[ \frac{\partial}{\partial P_1} [\Pi(P_1, P_2)] = \frac{\partial}{\partial P_2} [\Pi(P_1, P_2)] = 0 \]

\[ \frac{\partial}{\partial q_1} = \frac{\partial}{\partial q_2} \]

Case 1: \( q_1 > 0 \), goods 1 and 2 are substitutrs, \( \frac{P_1}{P_2} < \frac{C_1'}{C_2'} < \frac{1}{\frac{1}{c_{12}}} \)

Case 2: \( q_2 < 0 \), goods 1 and 2 are complements, \( \frac{P_1}{P_2} > \frac{1}{\frac{1}{C_{12}}} \)

Actually, both \( P_1 \) and \( P_2 \) are endogenous and have to be solved simultaneously.

\[ \left( 1 + \frac{P_{eq1}}{C_{1eq}} \right) \left( \frac{P_{eq2}}{P_2} \right) = \left( \frac{1}{C_{2eq}} \right) \]

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Masterbook of Business and Industry (MBI)

Muhammad Firman (University of Indonesia - Accounting)

4.4.1 2-period model with goodwill (Tirple EX 1.5)
Assume that $Q_2 = D_2(p_2|p_1, \gamma) = \frac{\partial D_2}{\partial p_1} < 0$, i.e., if $p_1$ is cheap, the monopoly gains goodwill in $t = 2$.

\[
\max_{p_1, p_2} D_1(p_1) - C(D_1(p_1)) + \delta D_2(p_2 - p_1) - C(D_2(p_2|p_1)).
\]

4.4.2 2-period model with learning by doing (Tirple EX 1.6)
Assume that $C_2 = C_2(p_2, Q_2)$ and $\frac{\partial C_2}{\partial Q_2} > 0$, i.e., if $Q_1$ is higher, the monopoly gains more experience in $t = 2$.

\[
\max_{p_1, p_2} D_1(p_1) - C(D_1(p_1)) + \delta D_2(p_2 - p_1) - C(D_2(p_2|p_1)).
\]

Continuous time (Tirple EX 1.7):

\[
\max_{Q_1} \int_0^T \left[ R(Q_0) - C(Q_0) e^{-\alpha dt} \right] dt, \quad Q_0 = 1 + \delta Q_1,
\]

where $R(Q_0)$ is the revenue at time $t$, $\alpha > 0$, $\delta > 0$, is the interest rate, $C(Q_0)$ is the unit production cost at $t$, $\alpha < 1$ and $Q_0$ is the experience accumulated at time $t$.

Example: $R(Q) = \sqrt{Q}$ and $C(Q) = a + \frac{1}{Q}$.

4.5 Durable good monopoly

Flow (perishable) goods:

Durable goods:

Cosco (1972): A durable good monopoly is essentially different from a perishable good monopoly.

Pereishable goods:

Durable goods:

4.5.1 A two-period model

There are 100 potential buyers of a durable good, say cars. The value of the service of a car to consumer $i$, at each period is $U_i = 101 - i$, $i = 1, \ldots, 100$.

Assume that $MC = 0$ and $0 < \delta < 1$ is the discount rate.

\[ U_i P \]

The trace of $U_i$’s becomes the demand curve.

$P^F$, $P^G$ are the prices in periods 1 and 2.

$P_1$, $P_2$ are the prices of a car in periods 1 and 2.

The monopoly faces the same demand function $P = 100 - Q$ in each period. The monopoly profit maximization implies that $MR = 100 - 2Q = 0$. Therefore,

\[ P^F = P^G = 50, \quad \pi^F = \pi^G = 2500, \quad \Pi^F = \pi^F + \delta \pi^G = 2500(1 + \delta), \]

where $\delta < 1$ is the discounting factor.

We use backward induction method to find the solution to the profit maximization problem. We first assume that those consumers who buy in period $t = 1$ do not resell their used cars to other consumers.

Suppose that $\gamma^F = \gamma^G = \gamma$, the demand in period $t = 2$ becomes

\[ \gamma^F = 100 - \gamma^F P_2 - \gamma^F Q_2 = 100 - \gamma^F - \gamma^F (100 - \gamma^F)/4. \]

Now we are going to calculate the location of the marginal consumer $Q_2$ who is indifferent between buying in $t = 1$ and buying in $t = 2$.

\[
(1 + \delta)\gamma(100 - \gamma) - P_2(50(1 - \gamma)) - P_1(100 - \gamma) = -\delta P_2(100 - \gamma - \gamma) + \delta P_2(100 - \gamma - \gamma),
\]

\[ \max P^F = -\delta \gamma = \frac{\delta}{1 + \delta} P^F(50(1 - \gamma)) = \frac{1}{1 + \delta} P^F(50(1 - \gamma)), \]

FOC: $q^F = 200/(4+5)$, $P^F = 50(2+5)/(4+5)$, $\Pi^F = 2500(2+5)/(4+5)$.

When a monopoly firm sells a durable good in $t = 1$ instead of leasing it, the monopoly loses some of its monopoly power, that is why $\Pi^F < \Pi^F$.

4.5.4 Consume problem

Sales in $t$ will reduce monopoly power in the future. Therefore, a rational expectation consumer will wait.

Consume conjecture (1972): In the zero horizon case, if $\delta > 1$ or $\alpha = 0$, then the monopoly profit $\Pi^F = 0$.

The conjecture was proved in different versions by Stokey (1981), Wilcox (1982), Gil, Sinnott, and Wilson (1986).

Tirple EX 1.8.

1. A monopoly is the only producer of a durable good in $t = 1, 2, 3, \ldots$. If $p_1, p_2, p_3, \ldots$ and $(\mu, p_1, p_2, \ldots)$ are the quantity and price sequences for the monopoly product, the profit is

\[ \Pi = \sum_{t=0}^{\infty} \delta^t \mu p_t. \]

2. There is a continuum of consumers indexed by $\alpha \in [0, 1]$, each needs 1 unit of the durable good.

\[ \mu = \alpha + \delta \mu + \delta^2 \mu + \ldots = \frac{\alpha}{1 - \delta}. \]

The utility of the durable good to consumer $\alpha$.

If consumer $\alpha$ purchases the good at $t$, his consumer surplus is

\[ \delta^t (\alpha - \mu) = \delta^t \frac{\alpha}{1 - \delta} - \mu. \]

3. A linear stationary equilibrium is a pair $(\lambda, \mu)$, $0 < \lambda, \mu < 1$, such that

(a) If $\alpha > \lambda$, then consumer $\alpha$ will buy in $t$ if he does not buy before $t$.

(b) If at $t$, all consumers with $\alpha > \lambda$ have purchased (not purchased) the durable good, then the monopoly chooses $p_2 = \mu$.

(c) The purchasing strategy of $(\alpha)$ maximizes consumer $\alpha$’s consumer surplus, given the pricing strategy $(b)$.

(d) The pricing strategy of $(b)$ maximizes the monopoly profits, given the purchasing strategy $(a)$.

The equilibrium is derived in Tirple as

\[ \lambda = \frac{1}{\sqrt{1 - \delta}}, \quad \mu = \sqrt{\lambda - \delta (1 - \delta)}/\delta, \quad \lim_{\alpha \to 0} \lambda = \infty, \quad \lim_{\alpha \to 1} \mu = 0. \]

One way a monopoly of a durable good can avoid the Consume problem is price commitment. By convincing the consumers that the price is not going to be reduced in the future, it can make the same amount of profit as in the rent case. However, the commitment equilibrium is not subgame perfect. Another way is to make the product has durable.

4.6 Product Selection, Quality, and Advertising

Tirple, CH2.

Product space, Vertical differentiation, Horizontal differentiation.

Good-Characteristics Approach, Hedonic prices.

Traditional Consumer-Theory Approach.

4.6.1 Product quality selection, Tirple 2.2.1, pp.100-4.

Inverse demand: $p = P(x, q)$, where $x$ is the quality of the product.

Total cost: $TC = C(q, q)$.

Social planner’s problem:

\[ \max_{q, x} W(q, x) = \int_0^1 P(x, q) dx - C(q, q). \]

FOC: $\frac{\partial W}{\partial q} = \int_0^1 P(x, q) dx - C(q, q) = 0$.

(1) $P > MC$,

(2) $\frac{1}{q} \int_0^1 P(x, q) dx = C(q, q)$. Averaged marginal valuation of quality should be equal to the marginal cost of quality per unit.

Monopoly profit maximization:

\[ \max_{q, x} W(q, x) = qP(x, q) - C(q, q). \]

FOC: $\Pi_q = qP(x, q) - C(q, q) = 0$. We maximize $\Pi_q = qP(x, q) - C(q, q)$. 


CHAPTER 5
BASIS OF GAME THEORY

In this part, we consider the situation when there are \( n > 1 \) persons with different objective (utility) functions, that is, different persons have different preferences over possible outcomes. There are two cases:

1. Game theory: The outcome depends on the behavior of all the persons involved. Each person has some control over the outcome; that is, each person controls certain strategic variables. Each one's utility depends on the decisions of all persons. We want to study how persons make decisions.

2. Public Choice: Persons have to make decisions collectively, e.g., by voting. We consider only game theory here.

Game theory: the study of conflict and cooperation between persons with different objective functions.

Example (a 3-person game): The accuracy of shooting at A, B, C is 1/3, 2/3, 1, respectively. Each person wishes to kill the other two to become the only survivor. They shoot in turn starting A.

Question: What is the best strategy for A?

5.1 Ingredients and classifications of games

A game is a collection of rules known to all players which determine what players may do and the outcomes and payoffs resulting from their choices.

The ingredients of a game:
1. Players: Persons having some influences upon possible income (decision makers).
2. Moves: decision points in the game at which players must make choices between alternative moves and randomization points (called nature's moves).
3. A play: A complete record of the choices made at moves by the players and realization of the randomization.
4. Outcomes and payoffs: a play results in an outcome, which in turn determines the records to players.

Classifications of games:
1. According to number of players:
   - 2-person games: conflict and cooperation possibilities.
   - \( n \)-person games: coalitions formation possibilities in addition.
2. According to number of strategies:
   - finite-strategy (matrix) games, each person has a finite number of strategies.
   - payoff functions can be represented by matrices.
   - infinite-strategy (continuous or discontinuous payoff functions) games like oligopoly games.
3. According to sum of payoffs:
   - \( 0 \)-sum games: conflict is unavoidable.
   - non-\( 0 \)-sum games: possibilities for cooperation.
4. According to play negotiation possibilities:
   - non-cooperative games: each person makes unilateral decisions.
   - cooperative games: players form coalitions and decide the redistribution of aggregate payoffs.

5.2 The extensive form and normal form of a game

Extensive form: The rules of a game can be represented by a game tree. The ingredients of a game tree are:

1. Players
2. Nodes: they are players' decision points (personal moves) and randomization points (nature's moves).
3. Information sets of player \( i \): each player's decision points are partitioned into information sets. An information set consists of decision points that player \( i \) cannot distinguish when making decisions.
4. Acts (choices): Every point in an information set should have the same number of choices.
5. Randomization probabilities (of acts following each randomization point).
6. Outcomes (and points).
7. Payoffs: The gains to players assigned to each outcome.

A pure strategy of player \( i \): An instruction that assigns a choice for each information set of player \( i \).

Total number of pure strategies of player \( i \): the product of the numbers of choices of all information sets of player \( i \).

Once we identify the pure strategy set of each player, we can represent the game in normal form (also called strategic form).

1. Strategy set for each player: \( S_i = \{ a_1, \ldots, a_n \} \), \( S_j = \{ b_1, \ldots, b_m \} \).
2. Payoff matrix: \( \pi_i(a_i, b_j) = a_{ij} \), \( \pi_j(a_i, b_j) = b_{ij} \). A = \( [a_{ij}] \), B = \( [b_{ij}] \).

Normal form:

\[
\begin{array}{c|ccc|}
\pi_j & a_1 & \cdots & a_n \\
\hline
b_1 & \pi_{1,1} & \cdots & \pi_{1,n} \\
\vdots & \vdots & \ddots & \vdots \\
b_m & \pi_{m,1} & \cdots & \pi_{m,n}
\end{array}
\]

5.3 Examples

Example 1: A perfect information game

\[
\begin{array}{c|cc|}
& L & R \\
\hline
I & 1 & 3 \\
& R & 2 & 1 \\
& R & 2 & 1 \\
& R & 2 & 1 \end{array}
\]

Example 2: Prisoner's dilemma game

\[
\begin{array}{c|cc|}
& L & R \\
\hline
L & (4,4) & (0,5) \\
R & (0,5) & (3,3) \\
R & (0,5) & (3,3) \end{array}
\]

Example 3: Hack game

\[
\begin{array}{c|cc|}
& L & R \\
\hline
L & (4,4) & (0,5) \\
R & (0,5) & (3,3) \end{array}
\]

Example 4: A simplified stock price manipulation game

\[
\begin{array}{c|ccc|}
& L & R \\
\hline
L & (4,4) & (0,5) \end{array}
\]

Remark: Each extensive form game corresponds to a normal form game. However, different extensive form games may have the same normal form.
5.4 Strategy pair and pure strategy Nash equilibrium

1. A strategy pair \((s_n, s_p)\). Given a strategy pair, there corresponds a payoff pair \((v_n, v_p)\).

2. A Nash equilibrium: A strategy pair \((s_n, s_p)\) such that \(v_n = a_{np} \geq a_{np}'\) and \(v_p = b_{np} \geq b_{np}'\) for all \((s, p)\). Therefore, there is no incentive for each player to deviate from the equilibrium strategy. \(a_{np}\) and \(b_{np}\) are called the equilibrium payoff.

The equilibrium payoffs of the examples are marked each with a star in the norm form.

Remark 1: It is possible that a game does not have a pure strategy Nash equilibrium. Also, a game can have more than one Nash equilibrium.

Remark 2: Notice that the concept of a Nash equilibrium is defined for a normal form game. For a game in extensive form (a game tree), we have to find the normal form before we can find the Nash equilibrium.

5.5 Subgame and subgame perfect Nash equilibria

1. Subgame: A subgame in a game tree is a part of the tree consisting of all the nodes and arcs following a node that form a game by itself.

2. Within an extensive game form, we can identify some subgames.

3. Also, each pure strategy of a player induces a pure strategy for every subgame.

4. Subgame perfect Nash equilibrium: A Nash equilibrium is called subgame perfect if it induces a Nash equilibrium strategy pair for every subgame.

5. Backward induction: To find a subgame perfect equilibrium, usually we work backward. We find Nash equilibria for lowest level (smallest) subgames and replace the subgame by its Nash equilibrium payoffs. In this way, the state of the game is reduced step by step until we end up with the equilibrium payoffs.

All the equilibria, except the equilibrium strategy pair (LIR) in the hijack game, are subgame perfect.

Remark: The concept of a subgame perfect Nash equilibrium is defined only for an extensive form game.

5.5.1 Perfect information game and Zermelo's Theorem

An extensive form game is called perfect information if every information set consists only one node. Every perfect information game has a pure strategy subgame perfect Nash Equilibrium.

5.5.2 Perfect recall game and Kuhn's Theorem

A local strategy at an information set \(u \in U\). A probability distribution over choice set at \(U\).

A behavior strategy: A function which assigns a local strategy for each \(u \in U\). The set of behavior strategies is a subset of the set of mixed strategies.

Kuhn's Theorem: In every extensive game with perfect recall, a strategically equivalent behavior strategy can be found for each behavioral strategy.

However, in a non-perfect recall game, a mixed strategy may do better than a behavioral strategy because in a behavior strategy the local strategies are independent whereas they can be correlated in a mixed strategy.

5.5.3 Reduction of a game

Redundant strategy: A pure strategy is redundant if it is strategically identical another strategy.

Reduced normal form: The normal form without redundant strategies.

Equivalent normal form: Two normal forms are equivalent if they have the same reduced normal form.

Equivalent extensive form: Two extensive forms are equivalent if their normal for are equivalent.

5.6 Continuous games and the duopoly game

In many applications, \(S_1\) and \(S_2\) are infinite subsets of \(R^m\) and \(R^p\). Player 1 controls \(m\) variables and player 2 controls \(n\) variables (however, each player has infinite many strategies). The normal form of a game is represented by two functions

\[ H^1 = H^1(x, y) \]

\[ H^2 = H^2(x, y) \]

where \(x \in S_1 \subset R^m\) and \(y \in S_2 \subset R^p\).

To simplify the presentation, assume that \(m = n = 1\). A strategy pair is \((x, y) \in S_1 \times S_2\). A Nash equilibrium is a pair \((x^*, y^*)\) such that

\[ H^1(x^*, y^*) \geq H^1(x, y) \\
H^2(x^*, y^*) \geq H^2(x, y) \]

for all \((x, y) \in S_1 \times S_2\). Consider the case when \(H^1\) and \(H^2\) are continuously differentiable and \(H^1\) and \(H^2\) are strictly concave in \(x\) and \(y\) so that we do not have to worry about the SOC's.

Reaction functions and Nash equilibrium:

To player 1, \(x\) is his endogenous variable and \(y\) is his exogenous variable. For each \(y\) chosen by player 2, player 1 will choose a \(x\) \(\in S_1\) to maximize his objective function \(H^1\). This relationship defines a behavioral equation \(x = R^1(y)\) which can be obtained by solving the FOC for player 1. \(H^1(x, y) = 0\). Similarly, player 2 regards \(y\) as endogenous and \(x\) as exogenous and wants to maximize \(H^2\) for a given \(x\) chosen by player 1. Player 2's reaction function (behavioral equation) \(y = R^2(x)\) is obtained by solving \(H^2(x, y) = 0\). A Nash equilibrium is an intersection of the two reaction functions. The FOC for a Nash equilibrium is given by \(H^1(x^*, y^*) = 0\) and \(H^2(x^*, y^*) = 0\).

Duopoly game:

There are two sellers (firm 1 and firm 2) of a product.

The (inverse) market demand function is \(P = a - Q\).

The marginal production costs are \(c_1\) and \(c_2\), respectively.

Assume that each firm regards the other firm's output as given (not affected by its output quantity).
The situation defines a 2-person game as follows: Each firm i controls its own output quantity $q_i$ (in million), to determine the market price $p = a - (c + q_1)$ which in turn determines the profits of each firm:

$$P_i = (a - c - q_i)a_i = a_i - c_i - (a - c - q_i)a_i$$

The POC are $\partial P_i/\partial q_i = a_i - c_i - q_i = 2 - q_i$ and $\partial P_i/\partial a_i = a_i - c_i = 2 - q_i$. The function is $a_i - c_i > 0$ and $a_i - c_i < 0$.

The Cournot Nash equilibrium is $\{q_1^*, q_2^*\} = \left(\frac{(a - 2c + 2d)^2}{2}, (a - 2c + 2d)^2/2\right)$ with $P_n = (c + 2c + c)^2/2$. (We have to assume that $a - 2c + 2d > 0$.)

### 5.7 2-person Nash game

1. Let $A = -A$ so that $q_i + q_j = 0$.

2. Maximin strategy: If player i plays $q_i$, then the minimum he will have is $\min q_j$, called the security level of strategy $q_i$. A possible rule for player 1 is to choose a strategy such that the security level is maximized: Player 1 chooses $q_1 = 0$, so that $\min q_j = 0$ for all $q_j$. Similarly, since $q_j = -q_i$, Player 2 also chooses $q_1 = 0$, so that $\max q_j = 0$ for all $q_j$.

3. Saddle point: If $q_1 = q_1^* = \max q_j$ and $q_2 = q_2^* = \min q_j$, then $(q_1^*, q_2^*)$ is called a saddle point. If a saddle point exists, then it is a Nash equilibrium.

### 4. Mixed strategy for player i:

A probability distribution over $S_i$, $p = (p_1, \ldots, p_m)$, $\sum p_i = 1$, is a mixed strategy $\pi$. Given $p_i$, $q_j$, the expected payoff of player 1 is $p_iq_j$. A mixed strategy Nash equilibrium $(\pi^*, q^*)$ is such that $p_j^* A_j^* p_i \geq p_j A_j^* p_i^*$ for all $j$ and $p_i \geq 0$.

### 5. Security level of a mixed strategy:

Given player i's strategy $\pi$, there is a pure strategy of player 2 so that the expected payoff to player 1 is minimized, just as in the case of a pure strategy of player i.

### 6. Linear programming problem:

The above problem can be transformed into a linear programming problem as follows: (a) Add a constant to each element of $A$ to insure that $t(p) > 0$ for all $p$ and $p_i > 0$. (b) Define $\gamma_i = \pi_i(t(p))$ and replace the problem of max $\gamma_i$ with the problem of min $t(p) = -\sum p_i i$. The constraints become $\sum p_i = 1$, $\gamma_i = 0$.

### 7. Duality:

It turns out that player 2's minimax problem can be transformed similarly and becomes the dual of player 1's linear programming problem. The existence of a mixed strategy Nash equilibrium is then proved by using the duality theorem in the linear programming problem.

### Example:

 Tos diving game: $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

To find player 2's equilibrium mixed strategy, we solve the linear programming problem:

$$\max_{x_1, x_2} \quad \text{subject to} \quad x_1 + x_2 = 1, x_1, x_2 \leq 1$$

The solution is $x_1 = x_2 = 1$ and therefore the equilibrium strategy for player 2 is $q_1^* = q_2^* = 0.5$.

Plauer 1's equilibrium mixed strategy is obtained by solving the dual to the linear programming problem:

$$\min c_1 + c_2 \quad \text{subject to} \quad \sum c_i = 1, c_i \geq 0$$

The solution is $p_1^* = p_2^* = 0.5$.

### mixed strategy equilibrium for non-zero sum games:

The idea of a mixed strategy equilibrium is also applicable to non-zero sum games. Similar to the simplex algorithm for the zero-sum game, there is a Lemke algorithm.

### Example (Game of Chicken)

**S**

<table>
<thead>
<tr>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front</td>
<td>3, 3</td>
</tr>
<tr>
<td>Back</td>
<td>-3, -3</td>
</tr>
</tbody>
</table>

There are two pure strategy NE: $\{S, N\}$ and $\{N, S\}$. There is also a mixed strategy NE: Suppose player 1 plays a mixed strategy $p \sim U(\{S, N\})$. If player 2 plays $S$, his expected payoff is $p(3, 3) = 3p + (-3)(1 - p)$. If player 2 plays $N$, his expected payoff is $p(3, 3) = 3p + (-3)(1 - p)$. For a mixed strategy NE, $p = \frac{1}{2}$.

The mixed strategy is symmetrical: $q_1^* = q_2^* = \left(\frac{1}{2}, \frac{1}{2}\right)$.

### 5.8 Cooperative Game and Characteristic form

2-person sum games are characteristically non-cooperative. If player 1 gains $S$ 1, player 2 will lose $S$ 1 and therefore no cooperation is possible. For other games, usually some cooperation is possible. The concept of a Nash equilibrium is defined for the situation when no explicit cooperation is allowed. In general, a Nash equilibrium is not efficient (not Pareto optimal).

When binding agreements on strategies chosen can be contracted before the play of the game and transfers of payoffs among players after a play of the game is possible, players will negotiate to coordinate their strategies and redistribute the payoffs to achieve better results. In such a situation, the determination of strategies is the key issue. The problem becomes the formation of coalitions and the distribution of payoffs.

### Characteristic form of a game:

The player set: $N = \{1, 2, \ldots, n\}$.

A coalition is a subset of $N$: $S \subseteq N$.

A characteristic function $v$ specifies the maximum total payoff of each coalition.

Consider the case of a 3-person game. There are 8 subsets of $N = \{1, 2, 3\}$, namely, $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$. Therefore, a characteristic form is determined by 8 values $v(S)$ for $S = \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$.

### Normal form:

Superadditivity: If $A \cap B = \emptyset$, then $v(A \cup B) \geq v(A) + v(B)$.

An impaction is a payoff distribution $(x_1, x_2, x_3)$.

### Individual rationality:

$v(S) \geq 0$.

### Core:

The set of imputations that are individually rational and group rationality for all $S$.

### Mammal contribution of player i in a coalition $S$:

$\phi_i(S) = v(S \cup i) - v(S)$.

### Shapley value of player i:

An average weighted average of all marginal contributions:

$$\phi_i(S) = \frac{1}{n!} \sum_{S_1 \subseteq S} |S_1| v(S \cup i) - v(S)$$

### Example:

$v(S) = v(1) = v(2) = v(3) = 0, v(12) = v(13) = v(23) = 5, v(123) = 13$.

$C = \{x_1, x_2, x_3\}$.

$(2, 3, 0, 0, 0, 0)$ are in $C$.

### Remark:

1. The core of a game can be empty. However, the Shapley values are uniquely determined.

2. A related concept is the von-Neumann-Morgenstern solution. See CH 6 of Inglehart's Mathematical Optimization and Economic Theory for the motivations of these concepts.

### 5.9 The Nash bargaining solution for a nontransferable 2-person cooperative game:

In a non-transferable cooperative game, after-play redistributions of payoffs are impossible and therefore the concepts of core and Shapley values are not applicable. For the case of 2-person games, the concept of Nash bargaining solutions are useful.

Let $F \subseteq P$. Be the feasible set of payoffs if the two players can reach an agreement and $T$ the payoff of player 1 if the negotiation breaks down. $T$ is called the threat point of player 1. The Nash bargaining solution $(\xi_1, \xi_2)$ is defined to be the solution to the following problem:

$$\min \xi_1, \xi_2 \quad \text{subject to} \quad \xi_1 \geq 1, \xi_2 \geq 1.$$
CHAPTER 6

DUOPOLY AND OLIGOPOLY – HOMOGENEOUS PRODUCTS

6.1 Cournot Market Structure

2 Sellers producing a homogeneous product.

\[ TC_i(q_i) = a_iq_i, \quad i = 1, 2. \]

\[ P(Q) = a - bQ, \quad a, b > 0, \quad a \geq \max_i a_i, \quad Q = q_1 + q_2. \]

Simultaneous move, both firms exercise \( q_1, q_2 \) simultaneously.

\[ \pi_i(q_1, q_2) = P(Q)q_i - c_iq_i = (a - bq_1 - bq_2)q_i - c_iq_i. \]

\[ \pi_i(q_1, q_2) = P(Q)q_i - c_iq_i = (a - bq_1 - bq_2 - c_i)q_i. \]

Definition of a Cournot equilibrium: \( (P^*, q_1^*, q_2^*) \) such that \( P^* = P(Q^*) = a - b(q_1^* + q_2^*) \) and

\[ \pi_1(q_1^*, q_2^*) \geq \pi_1(q_1, q_2^*), \quad \pi_2(q_1^*, q_2^*) \geq \pi_2(q_1^*, q_2), \quad \forall (q_1, q_2). \]

The first order conditions (FOC) are

\[ \frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0, \quad \frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - c_2 = 0. \]

In matrix form,

\[ \left( \begin{array}{c} \frac{2b}{3} \frac{b}{3} \\ \frac{a}{3} \frac{a}{3} \end{array} \right) \left( \begin{array}{c} q_1 \\ q_2 \end{array} \right) = \left( \begin{array}{c} a - c_1 \\ a - c_2 \end{array} \right), \quad \Rightarrow \left( \begin{array}{c} q_1^* \\ q_2^* \end{array} \right) = \frac{1}{3b} \left( \begin{array}{c} a - 2c_1 + c_2 \\ a - 2c_2 + c_1 \end{array} \right) \]

\[ Q^* = \frac{2a - c_1 + c_2}{3b}. \]

If \( c_1 \geq c_2 \) (say, due to R&D), then \( q_1^* \geq q_1^*, \quad q_2^* \geq q_2^*, \quad Q^* \geq Q^*, \quad P^* \leq P^*, \quad \pi_1 \leq \pi_1, \quad \pi_2 \leq \pi_2. \]

6.1.1 Reaction function and diagrammatic solution

From FOC, we can derive the reaction functions:

\[ q_1 = \frac{a - c_1}{2b} - 0.5Q \equiv R_1(q_2), \quad q_2 = \frac{a - c_2}{2b} - 0.5Q \equiv R_2(q_1). \]

6.1.2 \( N \)-seller case

\( N \) sellers, \( MC_i = c_i, \quad i = 1, \ldots, N, \quad P = P(Q) = a - bQ = a - b\sum_{j=1}^{N} q_j. \)

\[ \pi_i(q_1, \ldots, q_N) = P(Q)q_i - c_iq_i = (a - b\sum_{j=1}^{N} q_j)q_i - c_iq_i. \]

FOC is

\[ \frac{\partial \pi_i}{\partial q_i} = a - b\sum_{j=1}^{N} q_j - bq_i - c_i = 0, \quad \Rightarrow \quad \sum_{j=1}^{N} q_j = \frac{a - c_i}{b}. \]

\[ Q^* = \frac{N - \sum_{j=1}^{N} c_j}{N + 1}, \quad P^* = \frac{a + \sum_{j=1}^{N} c_j}{N + 1}, \quad \frac{\sum_{j=1}^{N} q_j}{N} = \frac{a + \sum_{j=1}^{N} c_j}{N + b(N + 1)}. \]

6.1.3 Welfare analysis for the symmetric case

Consumer surplus \( CS = \frac{(a - P)Q}{2} \), Social welfare \( W = CS + \sum_j \pi_j. \)

For the symmetric case, CS as functions of \( N \) are

\[ CS^*(N) = \frac{1}{2} \left( \frac{a + Nc}{N + 1} \right) \left( \frac{N}{b(N + 1)} \right)^2 \left( a - c \right)^2. \]

The sum of profits is

\[ \sum_j \pi_j = \sum_j \left( \frac{a - c}{b} \right) \frac{N}{N + 1} \left( a - c \right)^2, \quad lim_{N \to \infty} \sum_j \pi_j = 0. \]

6.2 Sequential moves – Stackelberg equilibrium

Consider now that the two firms move sequentially. At \( t = 1 \) firm 1 chooses \( q_1 \). At \( t = 2 \) firm 2 chooses \( q_2 \).


The consequence is that when choosing \( q_2 \), firm 2 already knows what \( q_1 \) is. On the other hand, in deciding the quantity \( q_1 \), firm 1 takes into consideration firm 2’s possible reaction, i.e., firm 1 assumes that \( q_2 = R_2(q_1) \). This is the idea of backward induction and the equilibrium derived is a subgame perfect Nash equilibrium.

At \( t = 1 \), firm 1 chooses \( q_1 \) to maximize the (expected) profit \( \pi_1 = \pi_1(q_1, R_2(q_1)) \):

\[ \max_{q_1} \left( a - b(q_1 + R_2(q_1)) - c_1 \right) \left( a - b(q_1 + R_2(q_1)) - c_2 \right) = \max_{q_1} \left( a - bq_1 - R_2(q_1) - c_1 \right) = \max_{q_1} \left( a - bq_1 - R_2(q_1) - c_2 \right). \]

The FOC (interior solution) is

\[ \frac{\partial \pi_1}{\partial q_1} = \frac{\partial \pi_1}{\partial R_2} + \frac{\partial \pi_1}{\partial c_1} = \frac{\partial \pi_1}{\partial c_2} = \frac{1}{2} \left( \frac{a - bq_1 - R_2(q_1) - c_1}{a - bq_1 - R_2(q_1) - c_2} \right) = 0, \]

\[ = q_1^* = \frac{a - 2c_1 + c_2}{2b}, \quad q_2^* = \frac{a + 2c_1 - 3c_2}{2b}, \quad Q^* = \frac{3a - 2c_1 - c_2}{3b} < Q^*, \quad P^* = \frac{a + 2c_1 - 3c_2}{4} < P^*, \quad \left( \frac{a + c_1 + c_2}{3} - P^* > \frac{a - 2c_1 + c_2}{2b} \right). \]

1. \( q_1^* = q_1^* \geq q_1^* \) depending on \( c_1, c_2 \).
2. \( q_1^* > q_1^* \) because \( q_1^* = \pi_1(q_1, R_2(q_1)) > \pi_1(q_1, R_2(q_1)) = \pi_1^* \).
3. \( q_1^* < q_1^* \) since \( q_1^* < q_1^* \) and \( P^* < P^*. \)
will choose a large enough \( c_0 \) to make market price zero. This is an incredible threat because firm 2 will hurt himself too. However, if firm 1 believes that the threat will be enforced, there can be all kind of equilibria.

6.2.2 Extension

The model can be extended in many ways. For example, when there are three firms choosing output quantities sequentially. Or firms 1 and 2 move simultaneously and firm 3 moves last, etc.

6.3 Conjecture Variation

In Cournot equilibrium, firms move simultaneously and, when making decision, expect that other firms will not change their quantities. In more general case, firms will form conjectures about other firms' behaviors.

\[
\sigma_1(q_1, q_2) = (a - b_1 q_1 - b_2 q_2) q_1, \quad \sigma_2(q_1, q_2) = (a - b_1 q_1 - b_2 q_2) q_2.
\]

The FOCs are

\[
\frac{\partial \pi_1}{\partial q_1} = a - b_1 q_1 - b_2 q_2 = 0, \quad \frac{\partial \pi_2}{\partial q_2} = a - b_1 q_1 - b_2 q_2 = 0.
\]

Assume that the conjectures are

\[
\frac{b_1 q_1}{b_2 q_2} = \lambda_1, \quad \frac{b_2 q_2}{b_1 q_1} = \lambda_2.
\]

In equilibrium, \( q_1 = \frac{a}{\lambda_1} \) and \( q_2 = \frac{a}{\lambda_2} \). The FOCs become

\[
a - b_1 \left( \frac{a}{\lambda_1} \right) - b_2 \left( \frac{a}{\lambda_2} \right) = 0, \quad a - b_1 \left( \frac{a}{\lambda_1} \right) - b_2 \left( \frac{a}{\lambda_2} \right) = 0.
\]

In matrix form,

\[
\begin{pmatrix}
2 + 3 & b \left( 2 + \lambda_1 \right) \\
3 + 2 \lambda_2 & a - b_2 \left( 2 + \lambda_2 \right) \\
\end{pmatrix}
\begin{pmatrix}
q_1 \\
q_2 \\
\end{pmatrix}
\begin{pmatrix}
a - c \lambda_1 \\
-a c \lambda_2 \\
\end{pmatrix}
\]

If \( \lambda_1 = \lambda_2 = 0 \), then it becomes the Cournot equilibrium.

6.3.1 Stackelberg Case: \( \lambda_1 = 0, \lambda_2 = R(q_2) = 0.5 \)

Assuming that \( c_1 = c_2 = 0 \).

\[
\begin{pmatrix}
q_1 \\
q_2 \\
\end{pmatrix}
\begin{pmatrix}
a \\
\frac{a}{\lambda_2/2} \\
\end{pmatrix}
\]

It is the Stackelberg leadership equilibrium with firm 1 as the leader. Similarly, if \( \lambda_1 = 0, \lambda_2 = R(q_1) = 0.5 \), then firm 2 becomes the leader.

6.3.2 Collusion case: \( \lambda_1 = q_1/q_1, \lambda_2 = q_2/q_1 \)

Assume that \( MC_1 = c_1 \) and \( MC_2 = c_2 \). The FOCs become

\[
\frac{a - b_1 (q_1 + q_2)}{c_2 - (q_1 + q_2)} = 0, \quad a - b_1 (q_1 + q_2) - c_2 (q_1 + q_2) = 0.
\]

That is, \( MR = MC = MC_2 \), the collusive solution.

The bias can be generalized to N-firm case.

6.4 Bertrand Price Competition

It is easier to change prices than to change quantities. Therefore, firms' strategic variables are more likely to be prices.

Bertrand model: firms determine prices simultaneously.

Question: In a homogeneous product market, given \( (p_1, p_2) \), how market demand is going to divide between firm 1 and firm 2?

Assumption: Consumers always choose to buy from the firm charging lower price. When two firms charge the same price, the market demand is divided equally between them. Let \( Q = D(p_1, p_2) \) be the market demand.

\[
Q = D(p_1, p_2) = \begin{cases}
0 & p_1 > p_2 \quad D(p_1, p_2) = \begin{cases}
0 & p_1 > p_2 \\
D(p_1) & p_1 < p_2
\end{cases} \\
D(p_2) & p_2 < p_1
\end{cases}
\]

Notice that individual firms' demand functions are discontinuous.

6.5 Price competition, capacity constraint, and Edgeworth Cycle

One way to resolve Bertrand paradox is to consider DRTS or increasing MC. In Bertrand price competition, if the marginal cost is increasing \( TC(q_i) > 0 \), then we have to consider the possibility of mixed strategy equilibria. See Wicklein's Supplement to Chapter 5. Here we discuss capacity constraint and Edgeworth cycle.

6.5.1 Edgeworth model

Assume that both firms have a capacity constraint \( q_1 \leq a \) and that the product is indivisible \( q_1 = a \). There are 3 consumers. Consumer \( i \) is willing to pay \( t = a \) dollars for one unit of the product, \( i = 1, 2, 3 \).

6.6 A 2-period model

At \( t = 1 \), both firms determine quantities \( q_1, q_2 \).

At \( t = 2 \), both firms determine prices \( p_1, p_2 \) after seeing \( q_1, q_2 \).

Demand function: \( P = 10 - Q \quad MC: c_1 = c_2 = 1 \).

Proposition: If \( q_1 + q_2 \leq 5 \), then \( p_1 = p_2 = a = 10 - (q_1 + q_2) \) is the equilibrium at \( t = 2 \).

Proof: Suppose \( p_1 = p_2 = a = 10 - (q_1 + q_2) \), then \( p_1 = p_2 \) maximizes firm 1's profit. The reasons are as follows.

1. If \( p_1 < p_2 \), then \( p_1 = p_2 \) is \( x_1 = x_2 \).
2. If \( p_1 > p_2 \), then \( p_1 = p_2 \) is \( x_1 = x_2 = 5 \).

\[
\pi_1(q_1, q_2) = q_1 (P - p_1) = (9 - q_1 - q_2), \quad \pi_2(q_1, q_2) = q_2 (P - p_2) = (9 - q_1 - q_2).
\]

Therefore, it becomes an authentic Cournot quantity competition game.

6.7 Infinite Repeated Game and Self-enforcing Collusion

Another way to resolve Bertrand paradox is to consider the infinite repeated version of the Bertrand game. To simplify the issue, assume that \( c_1 = c_2 = 0 \) and \( P = \min(1 - Q, 0) = \min(1 - q_1 - q_2, 0) \) in each period \( t \). The profit function of firm \( i \) at time \( t \) is
A pure strategy of the repeated game of firm $i$ is a sequence of functions $s_{it}$ of outcome history $H_{i,t}$:

$$s_{it} = (s_{it}(0), s_{it}(0), s_{it}(0), \ldots, s_{it}(H_{i,t}))$$

where $H_{i,t}$ is the history of a player of the repeated game up to time $t - 1$:

$$H_{i,t} = ((\omega_{1}, 0), \omega_{2}(0), \omega_{2}(1), \omega_{2}(1), \ldots, \omega_{2}(0), \omega_{2}(1))$$

and $\omega_{2}$ maps from the space of histories $H_{i,t}$ to the space of quantities $[a_{i} : 0 \leq a_{i} < a_{0}]$. Given a pair $(s_{it}, s_{jt})$, the payoff function of firm $i$ is

$$\Pi_{i}(s_{it}, s_{jt}) = \sum_{t=1}^{\infty} d_{t}(s_{it}(H_{i,t-1}), s_{jt}(H_{i,t-1})) = \sum_{t=1}^{\infty} d_{t}(s_{it}(0), s_{jt}(0))$$

Given the Cournot equilibrium $(q_{i}^{*}, q_{j}^{*}) = \left(\frac{1}{3}, \frac{1}{3}\right)$, we can define a Cournot strategy for the repeated game as follows:

$$s_{it}(H_{i,t-1}) = \left\{\begin{array}{ll}
q_{i}^{*} & \text{if } q_{i}(0) < 0.5Q_{m} \forall 0 \leq t \leq 1, 1, 2 \text{ otherwise}
\end{array}\right.$$  

It is straightforward to show that the pair $(s_{it}^{*}, s_{jt}^{*})$ is a Nash equilibrium for the repeated game.

### 6.7.1 Trigger strategy and tact colusive equilibrium

A trigger strategy $s^{T}$ for firm $i$ has a cooperative phase and a non-cooperative phase:

**Cooperative phase**

If both firms cooperate (choose quantity $q_{i} = q_{j} = 0.5Q_{m}$) up to period $t - 1$, then firm $i$ will cooperate at period $t$.

**Non-cooperative phase**

Once the cooperation phase breaks down (someone has chosen a different quantity), then firm $i$ will choose the Cournot equilibrium quantity $q_{i}^{*}$. If both firms choose the same strategy, then they will cooperate forever. If neither firm can benefit from changing to a different strategy, then $(s^{T}, s^{T})$ is a subgame-perfect Nash equilibrium.

Formally, a trigger strategy for firm $i$ is:

$$s^{T} = (s_{it}^{T}, s_{jt}^{T}, \ldots, s_{it}^{T}, \ldots)$$

$$s_{it}^{T} = \left\{\begin{array}{ll}
0.5Q_{m} & \text{if } q_{i}(t) < 0.5Q_{m} \forall 0 \leq t \leq 1, 1, 2 \text{ otherwise}
\end{array}\right.$$  

### 6.7.2 $(s^{P}, s^{T})$ is a SPNE for $\delta \geq \frac{9}{17}$

**Proof:**

1. At every period $t$ in the cooperative phase, if the opponent does not violate the cooperation, then firm $i$'s gain to continue cooperation is

$$\Pi^{C} = 0.55_{m} + 0.50_{m} + 0.50_{m} + \cdots = 0.55_{m}(1 + 0.5^{2} + \cdots) = \frac{1}{3}$$

If firm $i$ chooses to stop the cooperative phase, he will set $q_{i} = 0.5$ (the profit maximization output when $q_{i} = 0.25$) and then trigger the non-cooperative phase and gains the Cournot profit of 1.97 per period. Firm $i$'s gain will be

$$\Pi^{N} = \frac{9}{64} + \frac{9}{64} + \frac{9}{64} + \cdots = \frac{1}{17}(1 + 0.5^{2} + \cdots) = \frac{1}{17}.$$  

Therefore, the cooperative phase is the best strategy to continue cooperation.

2. In the non-cooperative phase, the Cournot quantity is the Nash equilibrium quantity in each period.

The cooperative phase is the equilibrium realization path. The non-cooperative phase is called off-equilibrium subgame.

### 6.7.3 Retaliation trigger strategy

A retaliation trigger strategy $s^{R}$ for firm $i$ has a cooperative phase and a retaliation phase:

**Cooperative phase**

The same as a trigger strategy.

**Retaliation phase**

Once the cooperation phase breaks down, then firm $i$ will choose the retaliation quantity $q_{i}^{*} = 1$ to make sure $P = 0$. If both firms choose the same strategy, then they will cooperate forever. If neither firm can benefit from changing to a different strategy, then $(s^{R}, s^{R})$ is a subgame-perfect Nash equilibrium.

Formally, a retaliation trigger strategy for firm $i$ is

$$s^{R} = (s_{it}^{R}, s_{jt}^{R}, \ldots, s_{it}^{R}, \ldots)$$

$$s_{it}^{R} = \left\{\begin{array}{ll}
0.5Q_{m} & \text{if } q_{i}(t) < 0.5Q_{m} \forall 0 \leq t \leq 1, 1, 2 \text{ otherwise}.
\end{array}\right.$$  

### 6.7.4 $(s^{P}, s^{R})$ is a NE for $\delta > \frac{1}{3}$

**Proof:**

At every period $t$ in the cooperative phase, if the opponent does not violate the cooperation, then firm $i$'s gain to continue cooperation is (same as the trigger strategy case)

$$\Pi^{C} = \frac{1}{3} - \delta$$

If firm $i$ chooses to stop the cooperative phase, he will set $q_{i} = \frac{3}{5}$ (same as the trigger strategy case) and then trigger the retaliation phase, making $0$ profit per period. Firm $i$'s gain will be

$$\Pi^{N} = \frac{9}{64} - \frac{9}{64} - \frac{9}{64} - \cdots = \frac{1}{17}(1 + 0.5^{2} + \cdots) = 0.$$  

Therefore, during the cooperative phase, the best strategy is to continue cooperation.

The retaliation phase is off-equilibrium subgame and never reached. Since the retaliation strategy is not optimal, the Nash equilibrium is the subgame-perfect.

In summary, if $\delta > \frac{9}{17}$, then the firms will continue to a SPNE; if $\frac{9}{17} > \delta > \frac{1}{3}$, then the firms will continue in a non-NE; if $\delta < \frac{1}{3}$, then the Nash equilibrium is impossible.

### 6.7.5 Folk Theorem of the infinite repeated game

In the above, we consider only the cooperation to divide the monopoly profit evenly. The same argument works for other kinds of distributions of monopoly profit or even aggregate profits less than the monopoly profit.

**Folk Theorem:** When $\delta \rightarrow 1$, every distribution of profits such that the average payoff per period $q_{i}^{T} \geq q_{i}^{*}$ can be implemented as a SPNE.

For sub-game non-perfect NEs, the individual profits can be even lower than the Cournot profit.

### 6.7.6 Finitely repeated game

If the monopoly phase is only repeated a finite time $t = 1, 2, \ldots, T$, we can use backward induction to find Subgame-perfect Nash equilibria. Since the last period $T$ is the same as a $t$-period monopoly game, both firms will play Cournot equilibrium quantity. Then, since the last period strategy is sure to be the Cournot quantity, it does not affect the choices at $t = T - 1$, therefore, at $t = T - 1$ both firms also play Cournot strategy. Similar argument is applied to $t = T - 2, t = T - 3$, etc., etc.

Therefore, the only possible SPNE is that both firms choose Cournot equilibrium quantity from the beginning to the end.

However, there may exist sub-game non-perfect NE.

### 6.7.7 Infinite repeated Bertrand price competition game

In the above, we have considered a repeated game of quantity competition. We can define an infinitely repeated price competition game and a trigger strategy similarly:

1. In the cooperative phase, a firm sets monopoly price and gains one-half of the monopoly profit $0.5p_{m} = 1/2$.
2. In the non-cooperative phase, a firm sets Bertrand competition price $p_{i} = 0$ and gains 0 profit.

To deviate from the cooperative phase, a firm obtains the whole monopoly profit $p_{m} = 1/4$ instantly. If $1/4(1 - \delta) > 1/4(1 - \delta) = 0.5$, the trigger strategy is a SPNE.

### 6.8 Drop in International Trade

#### 6.8.1 Reciprocal Dumping in International Trade

2 countries, $i = 1, 2$ each has a firm (also indexed by $i$) producing the same product. Assume that $MC = 0$, but the unit transportation cost is $c$. $q_{i}, q_{j}$ quantities produced by country $i$’s firm and sold in domestic market and foreign market, respectively.

$$q_{i} = q_{i}^{D} + q_{i}^{F}, q_{j} = q_{j}^{D} + q_{j}^{F}$$ aggregate quantities sold in countries 1 and 2’s market, respectively.
$P_i = a - bQ_i$: market demand in country $i$'s market.

The profits of the international duopoly firms are

$$
\Pi_i = P_iQ_i + (P_r - \tau)Q_i = [a - bQ_i + \frac{d}{3}][a - (\alpha + \delta + \gamma) - \tau]Q_i,
$$

$$
\Pi_0 = P_0Q_0 + (P_r - \tau)Q_0 = [a - bQ_0 + \frac{d}{3}][a - (\alpha + \delta + \gamma) - \tau]Q_0.
$$

Firm $i$ will choose $Q_i$ and $Q_f$ to maximize $\Pi_i$. The FOCs are:

$$
\frac{\partial \Pi_i}{\partial Q_i} = a - 2bQ_i - b\frac{\alpha + \gamma}{3}, \quad \frac{\partial \Pi_0}{\partial Q_0} = a - 2bQ_0 - b\frac{\alpha + \gamma}{3} - \tau = 0, \quad i = 1, 2.
$$

In a symmetric equilibrium $Q_1 = Q_2 = Q$ and $Q_f = Q - \frac{\alpha + \gamma}{3}$. In matrix form, the FOCs become:

$$
\begin{bmatrix}
2b & b \\
2b & a - \frac{\alpha + \gamma}{3} \\
\end{bmatrix}
\begin{bmatrix}
Q_i \\
Q_f \\
\end{bmatrix} = \begin{bmatrix}
a - \frac{\alpha + \gamma}{3} \\
\frac{\alpha + \gamma}{3} \\
\end{bmatrix}.
$$

$$
Q_f = Q - \frac{\alpha + \gamma}{3}.
$$

It seems that there is no dumping: the FOB price of exports $P_{\text{FOB}}$ is lower than the domestic price $P$,

$$
P_{\text{FOB}} - P = \frac{\alpha + \gamma}{3} > 0.
$$

However, since $P_{\text{FOB}} > M_0 - \beta$, there is no dumping in the MC definition of dumping.

The comparative statics with respect to $\tau$ is

$$
\frac{\partial \Pi_i}{\partial \tau} > 0, \quad \frac{\partial \Pi_f}{\partial \tau} > 0, \quad \frac{\partial \Pi_0}{\partial \tau} > 0.
$$

In this model, it seems that international trade is a waste of transportation costs and is unnecessary. However, if there is no international competition, each country's market would become a monopoly.

Extensions: 1. 2-stage game. 2. Comparison with monopoly.

6.8.2 Preferential Trade Agreement, Trade Creation, Trade Diversion

Free Trade Agreement FTA: Free trade among participants.

Customs Union CU: FTA plus uniform tariff rates towards non-participants.

Common Market CM: CU plus free factor mobility.

Consider the apple market in Taiwan.

**Demand:** $P = a - Q$

2 export countries: America and Japan. $P_1 < P_2$.

At $t = 0$, Taiwan imposes uniform tariff of $\frac{\tau}{2}$ per unit.

$$
P_1 = P_1 + \frac{\tau}{2}, Q_1 = a - P_1 - \frac{\tau}{2}, \quad W_1 = CS_1 + T_1 = \frac{aP_1}{2} + \frac{\tau}{2} - \frac{\tau^2}{2}.
$$

At $t = 1$, Taiwan and Japan form FTA, assume $P_1 < P_2 + \frac{\tau}{2}$.

$$
P_1 = P_1, \quad Q_1 = a - P_1, \quad W_1 = CS_1 + T_1 = \frac{aP_1}{2} - \frac{\tau^2}{2}.
$$

$$
W_1 - W_0 = 0.5(a - P_1)^2 + \frac{\tau^2}{2} - (a - P_1)(P_1 - P_1) = 0.
$$

From $a$ and $P_0$, FTA is more advantageous the higher $a$ and the lower $P_1$.

6.9 Duopoly Under Asymmetric Information

6.9.1 Incomplete Information Game

Incomplete information game: Some players do not completely know the rule of the game. In particular, a player does not know the payoff functions of other players. There are more than one type of a player, whose payoff function depends on his type. The type is known to the player himself but not to other players. There is a prior probability distribution of the type of a player.

Bayesian equilibrium (of a static game): Each type of a player is regarded as an independent player. Bayesian perfect equilibrium (of a dynamic game): In a Bayesian equilibrium, players will use Bayes' law to estimate the posterior distribution of the types of other players.

6.0.2 Modified Chicken Game

In the chicken dilemma game, assume that there are 2 types of player $2a$ and $2b$, $2a$ is as before but $2b$ is different, who prefers $NN$ to $SS$, $SS$ to being a chicken.

$$
\begin{bmatrix}
S & N \\
N & S \\
\end{bmatrix}
\begin{bmatrix}
a \\
0 \\
\end{bmatrix}
\begin{bmatrix}
S & N \\
N & S \\
\end{bmatrix}
\begin{bmatrix}
a \\
0 \\
\end{bmatrix}
$$

Asymmetric information: player 2 knows whether he is $2a$ or $2b$ but player 1 does not. However, player 1 knows that $\text{Pr}(2a) = \text{Pr}(2b) = 0.5$.

If we regard $2a$ and $2b$ as two different players, the game tree becomes:

$$
\begin{bmatrix}
S & N \\
N & S \\
\end{bmatrix}
\begin{bmatrix}
\text{SS} & \text{SN} \\
\text{NS} & \text{NN} \\
\end{bmatrix}
\begin{bmatrix}
a \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\text{SS} & \text{SN} \\
\text{NS} & \text{NN} \\
\end{bmatrix}
\begin{bmatrix}
a \\
0 \\
\end{bmatrix}
$$

In this incomplete information game, player 2 has a dominant strategy, $N$. Hence, it can be reduced to a 2-person game. The pure strategy Bayesian equilibrium is $N$ and $SS$. The mixed strategy equilibrium is different from the ordinary chicken game.

6.9.3 A duopoly model with unknown MC

2 firms, 1 and 2, with market demand $P = 2 - q_1 - q_2$.

$MC_1 = 1, \quad q_1 = q_1(1 - q_1 - q_2)$.

2 types of firm 2: $2a$ and $2b$.

$MC_{2a} = 1, \quad q_{2a} = q_{2a}(1 - q_1 - q_{2a})$.

$MC_{2b} = 0.5, \quad q_{2b} = q_{2b}(1 - q_1 - q_{2b})$.

Asymmetric information: firm 2 knows whether he is $2a$ or $2b$, but firm 1 does not. However, firm 1 knows that $\text{Pr}(2a) = \text{Pr}(2b) = 0.5$.

To find the Bayesian equilibrium, we regard the duopoly as a 3-person game with payoff functions:

$$
\Pi_1(q_1, q_{2a}, q_{2b}) = 0.5(1 - q_1 - q_{2a}) + 0.5q_1(1 - q_1 - q_{2b}),
$$

$$
\Pi_2(q_1, q_{2a}, q_{2b}) = 0.5q_{2a}(0.5 - q_1 - q_{2a}),
$$

$$
\Pi_3(q_1, q_{2a}, q_{2b}) = 0.5q_{2b}(0.5 - q_1 - q_{2b}).
$$

FOC are:

$$
0.5(1 - 2q_1 - q_{2a}) + 0.5(1 - 2q_1 - q_{2b}) = 0, \quad 0.5(1 - q_1 - 2q_{2a}) = 0, \quad 0.5(1 - q_1 - 2q_{2b}) = 0.
$$

The Bayesian equilibrium is $(q_1, q_{2a}, q_{2b}) = (0, 0, 0)$. 0.5(0.75 - q_1 - 2q_{2a}) = 0, 0.5(1 - q_1 - 2q_{2b}) = 0, 0.5(1 - q_1 - 2q_{2b}) = 0.
CHAPTER 7
DIFFERENTIATED PRODUCT MARKETS

7.1 2-Differentiated Products Duopoly

2 sellers producing differentiated products:

\[ p_i = a - \beta_i - \gamma_i \quad p_2 = a - \beta_2 - \gamma_2, \beta > 0, \beta^2 > \gamma^2. \]

In matrix form:

\[ \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} a \ -\beta \\ a \ -\gamma \end{pmatrix} \begin{pmatrix} \gamma \\ \gamma \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \]

where:

\[ a = \frac{a}{\beta - \gamma}, \quad \frac{\beta}{\beta - \gamma} = \frac{\gamma}{\beta - \gamma}. \]

If \( \gamma = 0 \), the firms are independent monopolists. If \( 0 < \gamma < \beta \), the products are substitutable. When \( \beta = \gamma \), the products are perfect substitutable (homogeneous).

\[ \text{Complements} \quad \text{Substitutes} \]

Define \( \delta = \frac{\beta^2}{a} \), degree of differentiation.

If \( \delta \) (hence \( \gamma, \beta > 0 \)), products are highly differentiated.

If \( \delta = 0 \) (hence \( \gamma, \beta = 0 \)), products are highly homogeneous.

Assume that \( T(C, q) = c(q) \), \( i = 1, 2 \). In a Cournot quantity competition duopoly game, the profits are represented as functions of \( (q_1, q_2) \):

\[ \pi_1(q_1, q_2) = (a - \beta_1 - \gamma_1)q_1 - (a - \beta_2 - \gamma_2)q_2, \]

\[ \pi_2(q_1, q_2) = (a - \beta_1 - \gamma_1)q_1 - (a - \beta_2 - \gamma_2)q_2. \]

In a Bertrand price competition duopoly game, the profits are represented as functions of \( (p_1, p_2) \):

\[ \pi_1(p_1, p_2) = (a - \beta_1 - \gamma_1)p_1 - (a - \beta_2 - \gamma_2)p_2, \]

\[ \pi_2(p_1, p_2) = (a - \beta_1 - \gamma_1)p_1 - (a - \beta_2 - \gamma_2)p_2. \]

It seems that Cournot game and Bertrand game are just a change of variables of each other. However, the Nash equilibrium is totally different. A change of variables of a game also changes its Nash equilibrium.

7.1.1 Change of variables of a game

Suppose we have a game in \( (x_1, x_2) \):

\[ \pi_1(x_1, x_2), \pi_2(x_1, x_2). \]

FOCs of the 2-game:

\[ \frac{\partial \pi_1}{\partial x_1} = 0, \quad \frac{\partial \pi_2}{\partial x_2} = 0, \quad (x_1^*, x_2^*). \]

Change of variables:

\[ x_1 = F(y_1, y_2), x_2 = G(y_1, y_2). \]

The profit functions for the 2-game is

\[ \pi_1(y_1, y_2) = \pi_2(F(y_1, y_2), G(y_1, y_2)). \]

FOCs of the 2-game:

\[ \frac{\partial \pi_1}{\partial y_1} = 0, \quad \frac{\partial \pi_2}{\partial y_2} = 0, \quad (y_1^*, y_2^*). \]

Because the FOCs for the 2-game is different from that of the 2-game, \( x_1 \neq F(y_1, y_2) \)
and \( x_2 \neq G(y_1, y_2) \) in general. Only when \( \delta = 0 \) \( x_1 = F(y_1, y_2) \) and \( x_2 = G(y_1, y_2) \).

In case of the duopoly game, a Bertrand equilibrium is different from a Cournot equilibrium in general. Only when \( \delta = 0 \) will the two equilibria the same. That is, if the two products are independent, the firms are independent monopolists and it does not matter whether we use prices or quantities as the strategic variables.

7.1.2 Quantity Game

Assume that \( c_1 = c_2 = 0 \). The payoff functions are

\[ \pi_1(q_1, q_2) = (a - \beta_1 - \gamma_1)q_1, \quad \pi_2(q_1, q_2) = (a - \beta_2 - \gamma_2)q_2. \]

The FOC of firm \( i \) and its reaction function are

\[ q - (\beta_i - \gamma_i)q = 0, \quad \Rightarrow q_i = R_i(q_{-i}) \]

\[ \frac{\partial q_i}{\partial q_{-i}} = \frac{\partial q_i}{\partial q_i} = \frac{\gamma_i}{\beta_i - \gamma_i} = \frac{\gamma_i}{\beta_i - \gamma_i}. \]

The larger \( \delta \), the steeper the reaction curve.

If products are independent, \( \delta = 0 \),

\[ q_i = 0. \]

In a symmetric equilibrium, \( q_1 = q_2 = q^*, q_1 = q_2 = q^* \).

\[ q^* = \frac{a}{\beta + \gamma}, \quad \pi^*(q^*) = \frac{a^2}{\beta + \gamma}. \]

Therefore, when the degree of differentiation increases, \( q^*, p^* \) and \( \pi^* \) will be increased. When \( \delta = 1 \), it reduces to the homogeneous case.

7.1.3 Price Game

Also assume that \( c_1 = c_2 = 0 \). The payoff functions are

\[ \pi_1(p_1, p_2) = (a - \beta_1 - \gamma_1)p_1, \quad \pi_2(p_1, p_2) = (a - \beta_2 - \gamma_2)p_2. \]

The FOC of firm \( i \) and its reaction function are

\[ p - (\beta_i + \gamma_i)p = 0, \quad \Rightarrow p_i = R_i(p_{-i}) \]

\[ \frac{\partial p_i}{\partial p_{-i}} = \frac{\partial p_i}{\partial p_i} = \frac{\gamma_i}{\beta_i + \gamma_i} = \frac{\gamma_i}{\beta_i + \gamma_i}. \]

The larger \( \delta \), the steeper the reaction curve.

If products are independent, \( \delta = 0 \),

\[ p_i = 0. \]

In a symmetric equilibrium, \( p_1 = p_2 = p^*, q_1 = q_2 = q^* \).

\[ p^* = \frac{a}{\beta + \gamma}, \quad \pi^*(p^*) = \frac{a^2}{\beta + \gamma}. \]

Therefore, when the degree of differentiation increases, \( p^*, q^* \) and \( \pi^* \) will be increased. When \( \delta = 1 \), it reduces to the homogeneous case and \( p^* = 0 \).

7.1.4 Comparison between quantity and price games

\[ p^* - q^* = \frac{\alpha}{4\delta + 1} > 0, \quad \Rightarrow p^* - q^* < 0. \]

As \( \delta = 0 \) and \( \alpha = 0 \), \( p^* = q^* \).

Strategic substitutes vs Strategic complements: In a continuous game, if the slopes of the reaction functions are negative as in the Cournot quantity game, the strategic variables (e.g., quantities) are said to be strategic substitutes. If the slopes are positive as in the Bertrand price game, the strategic variables (e.g., prices) are said to be strategic complements.
7.1.5 Sequential moves game

The Stackelberg quantity leadership model can be generalized to the differentiated product case. Here we use an example to illustrate the idea. Consider the following Bertrand game:

\[ q_1 = 168 - 2p_1 + q_2, \quad q_2 = 168 - 2q_1 + p_1; \quad TC = 0. \]

\[ \Rightarrow \pi_1 = R(q_1) - \frac{1}{2}q_1^2, \quad \pi_2 = R(q_2) - \frac{1}{2}q_2^2. \]

In the sequential game version, assume that firm 1 moves first:

\[ \pi_1 = 168 - 2p_1 + (42 + 0.25p_2); \quad \pi_2 = 112 + 0.25p_2. \]

FOC: 210 - 61p_1 = 0, \quad \Rightarrow \pi_1 = 60, \quad p_1 = 57, \quad q_1 = 165, \quad q_2 = 114, \quad \pi_2 = 6300, \quad \pi_3 = 6498.

\[ \Rightarrow \pi_2 > \pi_1, \quad \pi_3 > q_2, \quad \pi_3 > \pi_1 > \pi^* = 6272. \]

The Cournot game is:

\[ p_1 = 162 - \frac{2q_1 + q_2}{3}, \quad p_2 = 162 - \frac{2q_2 + q_1}{3}. \]

\[ \Rightarrow \pi_1 = R(q_1) - \frac{1}{2}q_1^2, \quad \pi_2 = R(q_2) - \frac{1}{2}q_2^2. \]

In the sequential game version, assume that firm 1 moves first:

\[ \pi_1 = 120 - 0.25q_1, \quad \pi_2 = 100.8, \quad \pi_3 = 67.2, \quad \pi^* = 6774. \]

FOC: 128 - \frac{7}{2}q_1 = 0, \quad \Rightarrow q_1 = 108, \quad q_2 = 81, \quad p_1 = 69, \quad p_2 = 78, \quad \pi_1 = 7452, \quad \pi_2 = 6318.

\[ \Rightarrow \pi_2 > \pi_1, \quad \pi_3 > q_2, \quad \pi_3 > \pi^* > 6774. \]

From the above example, we can see that in a quantity game firms prefer to be the leader whereas in a price game they prefer to be the follower.

7.2 Free entry/exit and LR equilibrium number of firms

So far we have assumed that the number of firms is fixed and entry/exit of new/exit firms is impossible. Now let us relax this assumption and consider the case when new/exit firms will enter/exit the industry if they can make a positive/negative profit. The number of firms is now an endogenous variable.

Assume that every existing and potential producer produces identical product and has the same cost function.

\[ TC(a) = F, \quad P = A - Q. \]

Assume that there are \( a \) firms in the industry. The Cournot oligopoly quantity competition equilibrium is (see A2.12):

\[ Q = \frac{A}{N + 1}, \quad \Rightarrow \pi_1 = \left( \frac{A}{N + 1} \right)^2 - F = \Pi(a). \]

If \( \Pi(N + 1) > 0 \), then at least a new firm will enter the industry. If \( \Pi(N) < 0 \), then some of the existing firms will exit. In LR equilibrium, the number of firms \( a^* \) will be such that \( \Pi(a^*) \geq 0 \Leftrightarrow \Pi(a^* + 1) < 0 \).

\[ \Pi(a^*) > 0 \quad \text{and} \quad \Pi(a^* + 1) < 0. \]

The model above can be modified to consider the case when firms produce differentiated products:

\[ p_i = A - q_i - \frac{\sum q_j}{2}, \quad \pi_i = \left( A - q_i - \frac{\sum q_j}{2} \right) q_i - F, \quad \Rightarrow \text{FOC: } A - 2q_i - \frac{\sum q_j}{2} = 0, \]

where \( 0 < \delta < 1 \). If there are \( N \) firms, in Cournot equilibrium, \( q_i = \phi \) for all \( i \), and

\[ q_i = \phi - \frac{A}{2(N - 1)}, \quad \Rightarrow \pi^* = \left( 1 + \frac{A}{2(N - 1)} \right)^2 - F. \]

The equilibrium number of firms is

\[ N^* = \frac{A}{\sqrt{F} + 1} - \frac{1}{\sqrt{F}} = \left[ \frac{1}{2} \left( \frac{A}{1 + F} \right) + 1 - \frac{1}{3} \right]. \]

When \( \delta = 1 \), it reduces to the homogeneous product case. When \( \delta \) decreases, \( N^* \) increases.

7.3 Monopolistic Competition in Differentiated Products

The free entry/exit model of last section assumes that firms compete in a oligopoly market, firms in the market are aware of the co-existent relationship. Now we consider a monopolistic competition market in which each firm regards itself as a monopoly firm.

7.3.1 Chamberlin Model

There are many small firms each produces differentiated product and has a negatively sloped demand curve. The demand curve of a typical firm is affected by the number of firms in the industry. If existing firms are making positive profits \( H > 0 \), new firms will enter making the demand curve shift down. Conversely, if existing firms are making negative profits \( H < 0 \), some of them will exit and the curve shifts up. In the industry equilibrium, firms are making zero profits; there is no incentive for entry or exit and the number of firms does not change.

7.3.2 Dixit-Stiglitz Model

Dixit-Stiglitz (1977) formulates Chamberlin model. There is a representative consumer who has \( I \) dollars to spend on all the brands available. The consumer has a CES utility function

\[ u(q_1, q_2, \ldots) = \left( \sum_{i=1}^{N} q_i^\alpha \right)^{1/\alpha} \]

\[ 0 < \alpha < 1. \]

The consumer’s optimization problem is

\[ \max_{q} \sum_{i=1}^{N} q_i^\alpha, \quad \text{subject to } \sum_{i=1}^{N} p_i q_i - I, \quad \Rightarrow E = \sum_{i=1}^{N} q_i^\alpha + \sum_{i=1}^{N} p_i q_i. \]

FOC is

\[ \frac{\partial L}{\partial q_i} = \alpha q_i^{\alpha - 1} - \lambda q_i = 0, \quad \Rightarrow q_i = \left( \frac{\lambda}{\alpha} \right)^{1/\alpha}. \]

As \( N \) is very large, \( \frac{\partial L}{\partial \lambda} \approx 0 \) and the demand elasticity is approximately \( q_i \approx 1 - \frac{1}{\alpha}. \)

The cost function of producer’s is \( TC(q_i) = (1-\alpha)q_i^{\alpha} - F - c_i. \)

\[ \max_{q_i} \pi_i = -c_i - TC(q_i) = A^\alpha q_i^{1/\alpha} - F - c_i. \]

The monopoly profit maximization pricing rule \( MC = P = (1-\frac{1}{\alpha})q_i \)

\[ q_i^* = \left( \frac{\lambda}{\alpha} \right)^{1/\alpha} \quad \Rightarrow N^* = \frac{1}{\alpha}. \]

In equilibrium, \( \pi = 0 \), we have (using the consumer’s budget constraint)

\[ q_i^* = A^{1/\alpha} \quad \Rightarrow N^* = \frac{1}{\alpha}. \]

The conclusions are:

1. \( p_i^* = c_i^*/\alpha \) and Lerner index of each firm is \( p_i^*/c_i = 1 - \frac{1}{\alpha}. \)

2. Each firm produces \( \phi^* = \frac{A^*}{\sqrt{F}} - \frac{1}{\sqrt{F}} \)

3. \( N^* = \frac{1}{\alpha}. \)

4. Variety effect:

\[ N^* = (N^* q_i^*)^{1/\alpha} = N^* + \frac{1}{\alpha} \Rightarrow q_i^* = (1-\alpha)q_i + \alpha q_i^{1-\alpha} - \frac{1}{\alpha}. \]

The size of the market is measured by \( I \). When \( I \) increases, \( N^* \) (the variety) increases proportionally. However, \( U^* \) increases more than proportionally.
7.3.3 Intra-industry trade

Hoeckner-Ohlin theory: A country exports products it has comparative advantage over other countries.

Intra-industry trade: In real world, we see countries export and import the same products, e.g., cars, wines, etc. It seems contradictory to Hoeckner-Ohlin’s comparative advantage theory.

Krugman (JJE 1979): If consumers have preferences for varieties, as in Dixit-Stiglitz monopolistic competition model, intra-industry trade is beneficial to trading countries.

No trade: $\rho = \alpha/\alpha, \sigma = (1-\alpha)/(\alpha)$, $U^* = \frac{(1-\alpha)^2}{\alpha^2}U^{\alpha_{1}}$.

Trade: $\rho = \alpha/\alpha, \sigma = (1-\alpha)/(\alpha)$, $U^* = \frac{(1-\alpha)^2}{\alpha^2}U^{\alpha_{1}}$.


7.4 Location Models
The models discussed so far assume that the product differentiation is exogenously determined. Location models provide a way to endogenize product differentiation.

7.4.1 Product characteristics
Different products are characterized by different characters.
Firms choose different product characteristics to differentiate their products.

Vertical differentiation: Consumers' preferences are consistent w.r.t. the differentiated character, e.g.,

Horizontal differentiation: Different consumers have different tastes w.r.t. the differentiated character, e.g.,

To determine the characteristics of a product is to locate the product on the space of product characteristics.

7.4.2 Hotelling linear city model
Assume that consumers in a market are distributed uniformly along a line of length $L$.

Firm A is located at point $a$, $P_A$ is the price of its product.

Firm B is located at point $L - b$, $P_B$ is the price of its product.

Each point $x \in [0, L]$ represents a consumer $x$. Each consumer demands a unit of the product, either purchases from A or B.

Pay: $U_x = -\frac{1}{2}(x - a)^2 - \frac{1}{2}(x - (L - b))^2$ if $x$ buys from A.

$\tau$ is the transportation cost per unit distance. $|x - a|$ is the distance between $x$ and A (B).

The marginal consumer $\hat{x}$ is indifferent between buying from A and B. The location of $\hat{x}$ is determined by

$$-P_A - \tau |x - a| - P_B - \tau |x - (L - b)| = \hat{x} - \frac{L + b + a}{2} - \frac{P_A - P_B}{2\tau}.$$ (1)

The location of $\hat{x}$ divides the market into two parts: $[0, \hat{x}]$ is firm A’s market share and $[\hat{x}, L]$ is firm B’s market share.

Therefore, given $(P_A, P_B)$, the demand functions of firms A and B are

$$D_A(P_A, P_B) = \hat{x} - \frac{L + b + a}{2} - \frac{P_A - P_B}{2\tau}, \quad D_B(P_A, P_B) = \frac{L - \hat{x} - \frac{L + b + a}{2} - \frac{P_A - P_B}{2\tau}}{2\tau}.$$ (2)

Assume that firms A and B engage in price competition and that the marginal costs are zero. The payoff functions are

$$\Pi_A(P_A, P_B) = P_A \left[ \frac{L + b + a}{2} - P_A - \frac{P_A - P_B}{2\tau} \right], \quad \Pi_B(P_A, P_B) = P_B \left[ \frac{L - \hat{x} - \frac{L + b + a}{2} - \frac{P_A - P_B}{2\tau}}{2\tau} \right].$$

The FOCs are (the SOC satisfied)

$$\frac{\partial \Pi_A}{\partial P_A} = \frac{L - b + a}{2} - 2P_A - P_B = 0, \quad \frac{\partial \Pi_B}{\partial P_B} = \frac{L - b + a}{2} - 2P_B - P_A = 0.$$ (3)

The equilibrium is given by

$$P_A = \frac{\tau (3L + b - a)}{6}, \quad P_B = \frac{\tau (3L + b - a)}{6}, \quad Q_A = \frac{3L - b}{6} - \frac{\tau (3L + b - a)}{6}, \quad Q_B = \frac{3L - b}{6} - \frac{\tau (3L + b - a)}{6}.$$ (4)

Horizontal differentiation is not the distance between the locations of A and B, the degree of product differentiation is measured by $\theta$. When $\theta$ or $\tau$ increases, the products are more differentiated, the competition is less intense, equilibrium prices are higher and firms are making more profits.

$$\frac{\partial \Pi_A}{\partial \theta} > 0, \quad \frac{\partial \Pi_B}{\partial \theta} > 0.$$ (5)

Moving towards the other firm will increase one’s profits. In approximation, the two firms will end up locating at the mid-point $\frac{L}{2}$. In this model the differentiation is minimized in equilibrium.

Note 1. In this model, we can not invert the demand function to define a country competition game because the Jacobian is singular.

2. The degree of homogeneity $\theta = \frac{1}{2}$ is not definable either. Here we use the distance between the locations of A and B as a measure of differentiation.

3. The result that firms in the Hotelling model choose to minimize product differentiation is so far only an approximation because the location of the marginal consumer $\hat{x}$ in (7) is not clearly defined. It is actually an upper-or continuous correspondence of $(P_A, P_B)$. The reaction functions are discontinuous and the price equilibrium does not exist when the two firms are too close to each other.

See Os Sany’s Appendix 7.6.
The true profit function of firm A is

\[ \Pi_A(P_A, P_B) = \begin{cases} \frac{P_A (L - b - a)}{2} & a < \hat{x} < L - b \\ \frac{P_B (L - a - b)}{2} & \hat{x} < a \\ \frac{P_A (L - a - b)}{2} & L - b \leq \hat{x} \end{cases} \]

where \( d = L - a - b \). Consider the case \( a = b, d = L - 2a \). The reaction function of firm A is (firm B's is similar):

\[ P_A = R_A(P_B) = \begin{cases} \frac{0.5 \tau (L + P_B)}{P_B - (L - 2a)} & a \leq \hat{x} \leq \frac{L - a + b}{2} \\ \frac{0.5 \tau (L + P_B)}{P_B - (L - 2a)} & \frac{L - a + b}{2} \leq \hat{x} \leq L \end{cases} \]

For \( a \leq \hat{x} \leq \frac{L}{4} \), the reaction functions intersect at \( P_A = P_B = \tau \hat{x} \). When \( a > \hat{x} / 4 \), the reaction functions do not intersect.

### 7.4.4 Quadratic transportation costs

Suppose now that the transportation cost is proportional to the square of the distance.

\[ \hat{P}_A = \begin{cases} -P_A - \tau (x - a)^2 & \text{if } \hat{x} \text{ buys from A} \\ -P_B - \tau (L - b)^2 & \text{if } \hat{x} \text{ buys from A} \end{cases} \]

The marginal consumer \( \hat{x} \) is defined by

\[ P_A - \tau (x - a)^2 = P_B - \tau (L - b)^2 \Rightarrow \hat{x} = \frac{L - b + a}{2} \]

The demand functions of firms A and B are

\[ D_A(P_A, P_B) = \hat{x} = \frac{0.5 \tau (L + P_B)}{L - 2a - \hat{x}} \quad D_B(P_A, P_B) = \hat{x} = \frac{0.5 \tau (L + P_B)}{L - 2a - \hat{x}} \]

The payoff functions are

\[ \Pi_A(P_A, P_B) = P_A \left( \frac{L - b + a}{2} \right) - \frac{P_A (L - a - b)}{2} \quad \Pi_B(P_A, P_B) = P_B \left( \frac{L - a + b}{2} \right) - \frac{P_B (L - a - b)}{2} \]

The FOCS are (the SCCs are satisfied)

\[ \frac{L - b + a}{2} \frac{2P_A - P_B}{L - 2a - \hat{x}} = 0, \quad \frac{L - a + b}{2} \frac{P_A - 2P_B}{L - 2a - \hat{x}} = 0. \]

The equilibrium is given by

\[ P_A = \frac{\tau (3L - 2a + b)}{6}(L - a - b) \quad P_B = \frac{\tau (3L - 2a + b)}{6}(L - a - b) \]

\[ \hat{P}_A = \frac{\tau (3L - 2a + b)}{6}(L - a - b) \quad \hat{P}_B = \frac{\tau (3L - 2a + b)}{6}(L - a - b) \]

Therefore, both firms will choose to maximize their distance. The result is opposite to the linear transportation case.

Two effects of increasing distance:
1. Increase the differentiation and reduce competition. II 1.
2. Reduce a firm's turf. II 1.

In the linear transportation case, the 1st effect dominates. In the quadratic transportation case, the 2nd effect dominates.

Also, \( P \) is differentiable in the quadratic case and the interior solution to the profit maximization problem is the global maximum.

### Welfare comparison:

- **Equilibrium transportation cost curve**
- **Social optimum transportation cost curve**

7.5 Circular Market Model

It is very difficult to generalize the linear city model to more than 2 firms. Alternatively, we can assume that consumers are uniformly distributed on the circumference of a round lake. We assume further that the length of the circumference is \( L \). First, we assume that there are \( N \) firms whose locations are also uniformly distributed along the circumference. Then we will find the equilibrium number of firms if entry/exit is allowed.

Firm \( i \) faces two competing neighbors, firms \( i - 1 \) and \( i + 1 \). Thus, (7.20) is modified as

\[ \Pi_i = (P_i - E) - \frac{c}{N} - \frac{c}{N} \]

As in the linear city model, each consumer needs 1 unit of the product. The utility of consumer \( x \), if he buys from firm \( j \), is

\[ U(x) = P_j - r(x - L_j) \quad j = i - 1, i + 1 \]

where \( L_j \) is the location of firm \( j \). To simplify, let \( L = 1 \). Given the prices \( P_i \) and \( P_{i+1} = P_{i-1} \), there are two marginal consumers \( \hat{x}_1 \) and \( \hat{x}_2 \) with

\[ \hat{x}_1 = \frac{P_i - P_{i-1}}{\tau} + \frac{1}{N}, \quad \hat{x}_2 = \frac{P_i - P_{i+1}}{\tau} + \frac{1}{N} \]

In equilibrium, \( P_i = P_{i+1} \)

\[ \Pi_i = \left( P_i - E \right) \left( \frac{P_i - P_{i+1}}{\tau} + \frac{1}{N} \right) \]

In a free entry/exit long run equilibrium, \( N \) is such that

\[ \Pi_i \geq 0, \quad \Pi_i + 1 \leq 0 \]

Equilibrium transportation cost:

\[ T(N) = \frac{N}{2N} + \frac{1}{2N} \]

Aggregate transportation cost:

\[ T(N) = N\left( \frac{1}{2N} + \frac{1}{2N} \right) \]

Aggregate Fixed Costs = \( NF \).

Since each consumer needs 1 unit, \( TVC = c \) is a constant. The total cost to the society is the sum of aggregate transportation cost, TVC, and fixed cost.

\[ SC(N) = T(N) + c + NF \]

The conclusion is that the equilibrium number of firms is twice the social optimum number.

Remark: 1. AC is decreasing (DRTS), low firms will save production cost.
2. TVC is decreasing with \( N \), more firms will have consumers’ transportation cost.
3. The monopolistic competition equilibrium ends up with too many firms.

7.6 Sequential entry in the linear city model

If firm 1 chooses his location before firm 2, then clearly firm 1 will choose \( l_1 = l/2 \) and firm 2 will choose \( l_2 = l/2 \). If there are more than two firms, the Nash equilibrium location choices become much more complicated.

Assume that there are \( N \) firms and they enter the market (select locations) sequentially. That is, firm 1 chooses \( x_1 \), then firm 2 chooses \( x_2 \), and finally firm \( N \) chooses \( x_N \). To simplify, we assume that firms choose the same price \( p \) and that \( x_1 = 0 \). We want to find the equilibrium locations \( x_2 \) and \( x_N \).

1. If firm 2 chooses \( x_2 \) and \( x_N \) are

\[ x_2 = \frac{1}{2} \]


2. If firm 2 chooses \( x_2 \) and \( x_N \) are

\[ x_2 = \frac{1}{2} \]

\[ x_N = \frac{1}{2} \]

\[ x_N = \frac{1}{2} \]
CHAPTER 8
CONCENTRATION, MERGERS, AND ENTRY BARRIERS

8.1 Concentration Measures

We want to define some measures of the degree of concentration of an industry in order to compare different industries or a similar industry in different countries.

\[ i = 1, 2, \ldots, N, \quad Q_i = q_1 + q_2 + \cdots + q_N. \]

Market shares: \( q_i = \frac{Q_i}{Q} \times 100\% \), \( q_1 \geq q_2 \geq q_3 \geq \cdots \geq q_N. \)

Industry concentration index: \( I_2 = \frac{\sum_{i=1}^{N} q_i}{n} \).

Heron-Index-Hirschman index: \( I_{HH} = \frac{\sum_{i=1}^{N} q_i^2}{q_Y} \).

Clint coefficient: \( C = \frac{1}{N} \sum_{i=1}^{N} q_i^2 - \left( \frac{1}{N} \sum_{i=1}^{N} q_i \right)^2. \)

Entropy: \( I_2 = -\sum_{i=1}^{N} q_i \ln q_i. \)

8.1.1 Relationship between \( I_{HH} \) and Lerner index

Consider a quantity competition oligopoly industry. \( P(Q) = P(q_1 + q_2 + \cdots + q_N) \) is the market demand. \( \lambda_i \) is the conjecture variation of firm \( i \), i.e., when firm \( i \) increases 1 unit of output, he expects that all other firms together will respond by increasing \( \lambda_i \) units of output.

\[ \tau_i = P(Q) - q_i \lambda_i, \quad Q_i = \sum_{j \neq i} q_j \lambda_j, \quad \lambda_i = \frac{dP(Q)}{dq_i}. \]

\[ \frac{\partial P}{\partial q_i} = \frac{\partial P}{\partial q_i} \frac{\partial q_i}{\partial q} = -\frac{q_i P(Q)}{P(Q) + \lambda_i} = \frac{q_i P(Q)}{P(Q) + \lambda_i}. \]

\[ L_i = \frac{P}{P(Q)} - \frac{q_i P(Q)}{P(Q) + \lambda_i} = \frac{P - \frac{q_i P(Q)}{P(Q) + \lambda_i}}{P(Q)} = \frac{1}{q_i P(Q)} \left( \frac{q_i P(Q) + \lambda_i}{P(Q)} - 1 \right) \]

Industry average Lerner index: \( L = \frac{1}{N} \sum_{i=1}^{N} L_i = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{q_i P(Q)}{P(Q)} + \frac{1}{q_i P(Q)} \right) - 1 \)

If \( \lambda_i = \lambda \) for all \( i \), then \( L = \frac{\text{MR} - \text{MC}}{\text{MR}} \).

The ease of collusion \( \lambda_i = \frac{Q_i}{q_i} \).

8.2 Mergers

Mergers, takeovers, acquisitions, integration.

3 types:
Horizontal mergers: between the same industry
Vertical mergers: between upstream industry firms and down steam industry firms
Conglomerate mergers: other cases.

In US economic history there were 4 active periods:
1901: mostly horizontal and vertical
1920, 1960, 1980: other types, mostly influenced by changes in anti-trust laws.

Purpose: (1) reduce competition, (2) HRSt., (3) differences in the prospective of firms between sellers and buyers, (4) managers’ intention to enlarge their own careers, (5) the interests of the promoters.

8.2.1 Horizontal merger

\[ \text{concentration ratio} \downarrow \rightarrow \text{competition} \downarrow \rightarrow \text{welfare} \uparrow \]

Not necessary. If high cost (marginal) firms are taken-over, efficiency increases.

Example: In Cournot duopoly model, assume \( c_1 = 1, c_2 = 2, P = 10 - Q \).

\[ q_1^* = 4, q_2^* = 1, P^* = 5, \quad \tau_1 = 16, \quad \tau_2 = 1, \quad CS = 125, \quad \text{W} = 29.5. \]
If firms 1 and 2 merge to become a monopoly, the monopoly would shut down the production of firm 2 so that the MC of the monopoly is \( e = 1 \).

\[
Q^M = 4.5, \quad P^M = 5.5, \quad \pi^M = 20.25, \quad CS = 10.125, \quad W^M = 30.375 > W^*.
\]

Comparison of Welfare: \( W^M > W^* \), it is the trade-off between the production efficiency and monopoly inefficiency.

Comparison of \( I_{nu} \): \( I_{nue} = 6, 800, \quad I_{nu} = 10, 000 \).

According to Anti-trust law, such a merger is prohibited, but it is good to the society. However, if the market is originally in Bertrand price competition, the conclusion is totally different. In a Bertrand competition market, the industry is efficient.

8.2.2 Vertical merger

If both upstream and downstream industries produce homogeneous products and are Bertrand price competition markets, the merger does not affect anything.

Assumption: The upstream was originally in Bertrand competition and the downstream was in Cournot competition.

Downstream market demand: \( P = \alpha - q_1 - q_2 \), \( MC = \alpha \), \( q_1 = q_2 = \alpha \).

\[
\Rightarrow q_1 = q_2 = \frac{\alpha - 2a + c_2}{3}, \quad \pi_1 = \frac{(\alpha - 2a + c_2)^2}{9}, \quad q_2 = \frac{\alpha - 2a + c_2}{3}, \quad P = \alpha - \frac{\alpha + c_1 + c_2}{3}
\]

Upstream: Assume \( MC = MC_2 = 0 \), Bertrand equilibrium: \( p_u = p_m = e_1 = c_1 = 0 \).

Post-merger Equilibrium:

\[
q_1 = q_2 = \frac{\alpha}{3}, \quad \pi_1 = \frac{\alpha^2}{3}, \quad \pi_2 = \frac{\alpha^2}{3}, \quad P = \frac{\alpha}{3}, \quad Q = \frac{\alpha^2}{3}
\]

Post-merger: Assume that A1 does not sell raw material to 2. B becomes an upstream monopoly. We ignore the fact that 2 is also downstream monopoly:

\[
x = \frac{\alpha(\alpha - 2a + c_2)}{9}, \quad p_m = \frac{\alpha(\alpha - 2a + c_2)}{3}, \quad \max \pi_g = \pi_m = \pi_2 = \frac{\alpha^2}{3}
\]

The effects of merger: \( P_1 \), \( q_1 \), \( q_2 \), \( q_1 + q_2 \), \( q_1 + q_2 + q_3 \), \( q_1 + q_2 + q_3 + q_4 \).

8.2.3 merger of firms producing complementary goods

Firm X produces PCs and Firm Y produces monitors. A system is \( X + Y \). \( P_x = P_y = P_y \).

Market demand: \( Q_x = \alpha - P_x = \alpha - P_x - P_y, \quad Q_y = Q_y, \quad Q_x = Q_y + Q_y \).

Pre-merger:

\[
\Pi_x = P_xQ_x = P_x(\alpha - P_x - P_y), \quad \Pi_y = P_yQ_y = P_y(\alpha - P_x - P_y).
\]

\[
\alpha - 2P_x - P_y = 0, \quad P_x = P_y - \frac{\alpha^2}{3}, \quad \Pi_x = \Pi_y = \frac{\alpha^2}{9}, \quad P_x = P_y = \frac{\alpha}{3}, \quad Q_x = Q_y = \frac{\alpha}{3}, \quad P_x = P_y = \frac{\alpha}{3}
\]

Post-merger:

\[
\Pi_x = P_xQ_x = P_x(\alpha - P_x), \quad P_x = \frac{\alpha}{2}, \quad \Pi_x = \frac{\alpha^2}{4}
\]

The effects of merger:

\[
P_x = \frac{\alpha}{3}, \quad P_2 = P_y = \frac{2\alpha}{3}, \quad Q_x = \frac{\alpha}{3}, \quad Q_y = \frac{\alpha}{3}, \quad \Pi_x = \frac{\alpha^2}{4} \quad \Pi_x + \Pi_y = \frac{2\alpha^2}{4}
\]

Therefore, one monopoly is better than two monopolies.

Remarks: 1. It is similar to the joint product monopoly situation. When a monopoly produces two complementary goods, the profit percentage should be lower than individual Lerner indices. In this case, a higher \( P_x \) will reduce the demand for \( Y \) and vice versa. After merger, the new firm internalizes these effects. 2. The model here is isomorphic to the Cournot duopoly model with price variables and quantity variables interchanged.

8.3 Entry Barriers

1. Economy of scale, large fixed cost
2. Production differentiation advantages (reputation, goodwill)
3. Consumer loyalty, network externalities
4. Absolute cost advantages (learning experiences)
5. Location advantage (sequential entry)
6. Other advantages

Incumbents may also take entry deterrence strategies.

8.3.1 Fixed cost and \( I_{nu} \)

For Dixit-Stiglitz monopolistic competition model, \( N = \frac{(1 - \alpha)F}{P} \):

\[
I_{nu} = N \left( \frac{100}{N} \right)^2 = 10, 000, \quad F = \frac{100}{N}, \quad \frac{\partial I_{nu}}{\partial F} > 0.
\]

In quantity competition with free entry/exit model, \( N = \frac{(1 - \alpha)^3}{\sqrt{F} I_{nu}} \):

\[
I_{nu} = \frac{10, 000}{N} = \frac{100}{N} \frac{10, 000}{I_{nu}}, \quad \frac{\partial I_{nu}}{\partial F} > 0.
\]

In the circular city model, \( N = \sqrt{\frac{F}{I_{nu}} - \frac{F}{I_{nu}}}, \quad \frac{\partial I_{nu}}{\partial F} < 0 \).

8.3.2 Sunk cost

Sunk costs: Costs that cannot be reversed. Firm-specific equipment, etc.

Sunk costs:

\[ A: \text{An Incumbent}, \quad B: \text{A Potential Entrant} \]

\[ \Pi^A = \begin{cases} \Pi^M - \epsilon & \text{if no entry} \quad \Pi^B = \begin{cases} 0 & \text{do not enter} \quad \epsilon & \text{B enters} \end{cases} \end{cases} \]

where \( \Pi^M \) is the monopoly profit and \( \epsilon \) is the sunk cost.

\[ \begin{cases} \text{Exit} \quad \text{Stay out} \end{cases} \]

Proposition: As long as \( 0 < \epsilon < \Pi^M \), there exists only one subgame perfect equilibrium, i.e., \( B \) stays out.

Condition: 1. A and B produce homogeneous products with identical marginal cost.
2. Post-entry market is a Bertrand duopoly.
3. A cannot retreat.

If B can invest in product differentiation to avoid homogeneous products Bertrand competition or the post-entry market is a Cournot duopoly, then the proportion is not valid anymore.

If A can resale some of its investments, say, recover \( \phi > 0 \). The game becomes

\[ \begin{cases} \text{Exit} \quad \text{Stay out} \end{cases} \]

\[ \begin{cases} \epsilon & \Pi^M - \epsilon \end{cases} \]

Therefore, one monopoly is better than two monopolies.
The only subgame-perfect equilibrium is that B enters and A exits. However, the result is just a new monopoly replacing an old one. The sunk cost c can be regarded as the entry barrier as before.
Notice that there is a non-perfect equilibrium in which A chooses the incredible threat strategy of stay in and B chooses stay out.

8.4 Entry Deterrence

8.4.1 Burning one’s bridge strategy (Tirole Chs)

Two countries wishing to occupy an island located between their countries and connected by a bridge to both. Each army prefers letting its opponent have the island to fighting. Army 1 occupies the island and burns the bridge behind it. This is the paradox of commitment.

\[
\begin{align*}
\text{Arm 1} & \quad \text{Arm 2} \\
\text{Island} & \quad \text{Island} \\
\text{Bridge} & \quad \text{Bridge}
\end{align*}
\]

8.4.2 Simultaneous vs. Sequential Games

Consider a 2-person game:
\[
\Pi_1(x_1, x_2; y_1, y_2), \quad \Pi_2(x_1, x_2; y_1, y_2),
\]
where \((x_1, y_1)\) is firm 1’s strategy variable and \((x_2, y_2)\) is firm 2’s strategy variable.

Simultaneous game: Both firms choose \((x, y)\) simultaneously.

FOC: \[
\frac{\partial \Pi_1}{\partial x_1} = 0, \quad \frac{\partial \Pi_1}{\partial y_1} = 0,
\]
\[
\frac{\partial \Pi_2}{\partial x_2} = 0, \quad \frac{\partial \Pi_2}{\partial y_2} = 0.
\]

Sequential game: In \(t = 1\) both firms choose \(x_1\) and \(x_2\) simultaneously and then in \(t = 2\) both firms choose \(y_1\) and \(y_2\) simultaneously.

To find a subgame perfect equilibrium, we solve backward:

\[
\begin{align*}
\text{In } t = 2, \quad x_2 \text{ and } x_2 \text{ are given, the FOC are:} & \quad \frac{\partial \Pi_1}{\partial x_1} = 0 \quad \text{and} \quad \frac{\partial \Pi_2}{\partial y_2} = 0.
\end{align*}
\]

In \(t = 1\), the reduced game is:

\[
\begin{align*}
\pi_1(x_1, x_2; y_1, y_2) = \Pi_1(x_1, x_2, f(x_1, x_2), g(x_1, x_2)), \quad \pi_2(x_1, x_2) = \Pi_2(x_1, x_2, f(x_1, x_2), g(x_1, x_2)).
\end{align*}
\]

The FOC is:

\[
\frac{\partial \Pi_1}{\partial x_1} + \frac{\partial \Pi_2}{\partial y_1} = 0, \quad \text{and} \quad \frac{\partial \Pi_1}{\partial x_2} + \frac{\partial \Pi_2}{\partial y_2} = 0.
\]

Since \(\frac{\partial \Pi_1}{\partial x_2} = 0\), the FOC becomes:

\[
\frac{\partial \Pi_1}{\partial x_1} + \frac{\partial \Pi_2}{\partial y_1} = 0, \quad \text{and} \quad \frac{\partial \Pi_1}{\partial x_2} = 0.
\]

Compare the FOC of the sequential game with that of the simultaneous game, we can see that the equilibria are not the same. In \(t = 1\), both firms try to influence the \(t = 2\) decisions of the other firms. For the simultaneous game, there is no such consideration.

\[
\frac{\partial \Pi_1}{\partial x_1} \quad \text{and} \quad \frac{\partial \Pi_2}{\partial y_2}
\]

are the strategic consideration terms.

Entry deterrence application: In \(t = 1\), only firm 1 exists.

\[
\Pi_1(x_1, x_1; y_1, y_2), \quad \Pi_2(x_1, x_1; y_1, y_2).
\]

Firm 1 is the incumbent and tries to influence the entry decision of firm 2.

8.4.3 Spence (1977) entry deterrence model

In this model the incumbent attempts to use strategic capacity investment to deter the entry of a potential entrant.

\[
t = 1: \quad \text{Firm 1} \text{ decides its capacity-output level } k_1.
\]

\[
t = 2: \quad \text{Firm 2} \text{ decides its capacity-output level } k_2.
\]

If \(k_2 > 0\), firm 2 enters. If \(k_2 = 0\), firm 2 stays out.

In \(t = 1\), firm 1 takes into consideration firm 2’s discontinuous reaction function. If \(1 - 2\sqrt{E} \leq k_2 \leq 1\), then firm 1 will choose monopoly output \(k_1 = \frac{1}{2}\) and firm 2 will stay out. This is the case of entry blocked.

Next, we consider the case \(E \leq \frac{1}{4}\). If firm 1 chooses monopoly output, firm 2 will enter. Firm 1 is considering whether to choose \(k_1 > 1 - 2\sqrt{E}\) to force firm 2 to give up or to choose \(k_1 < 1 - 2\sqrt{E}\) and maximize duopoly profits.

1. entry deterrence: If firm 2 stays out (\(k_2 > 1 - 2\sqrt{E}\) and \(k_2 = 0\)), firm 1 is a monopoly by deterrence.

\[
\pi_1^E = \max_{k_1 < 1 - 2\sqrt{E}} \pi_1(k_1, k_2) = \pi_1^D(k_1, 1 - k_1), \quad \Rightarrow \pi_1^E = \frac{k_1^2}{2} (1 - k_1^2), \quad \text{where } k_1 = 1 - 2\sqrt{E}.
\]

2. entry accommodation: If firm 2 enters (\(k_2 < 1 - 2\sqrt{E}\) and \(k_2 > 0\)), firm 1 is the leader of the Stackelberg game.

\[
\pi_1^E = \max_{k_1 < 1 - 2\sqrt{E}} \pi_1(k_1, k_2) = \frac{k_1^2}{2} (1 - k_1^2), \quad \Rightarrow \pi_1^E = \frac{k_1^2}{2} (1 - k_1^2), \quad \text{where } k_1 = 1 - 2\sqrt{E}.
\]

The Stackelberg game is:

\[
\begin{align*}
&\text{Entry blocked} \quad k_2 = 0 \quad \text{or} \quad k_2 \geq 1 - 2\sqrt{E}, \\
&\text{Entry deterred} \quad k_2 < 1 - 2\sqrt{E}, \quad k_2 > 0, \quad \text{and} \quad \pi_1^E = \pi_1^E(k_1, k_2) \geq \pi_1^E(k_1, 1 - k_1), \quad \text{where} \quad k_1 = 1 - 2\sqrt{E}.
\end{align*}
\]

Entry: firm 1 and firm 2 compete for market share. If \(E > \frac{1}{16}\), firm 1 is a monopoly and firm 2’s entry is blocked.

Entry blocked: firm 1 and firm 2 compete for market share. If \(E < \frac{1}{16}\), firm 1 will choose \(k_1 > k_2\) to deter firm 2’s entry; if \(E < \frac{1}{16}\), firm 1 will choose \(k_1 < k_2\) to deter firm 2’s entry.

Entry deferred: if \(E = \frac{1}{16}\), firm 1 will accommodate firm 2’s entry; if \(E < \frac{1}{16}\), firm 1 will choose \(k_1 < k_2\) to deter firm 2’s entry.

In summary, if \(E < \frac{1}{16}\), firm 1 will accommodate firm 2’s entry; if \(E > \frac{1}{16}\), firm 1 is a monopoly and firm 2’s entry is blocked.

Spence model is built on the so-called Bain-Sylva style assumption:
1. Firm 2 (entrant) believes that firm 1 (incumbent) will produce \(k_1 - k_2\) after firm 2’s entry is deterred. However, it is not optimal for firm 1 to do so. Therefore, the equilibrium is not subgame perfect.
2. Firm 2 has sunk costs but not firm 1. The model is not symmetric.

8.4.4 Friedman and Dixit’s criticism

1. Incumbent’s pre-entry investment should have no effects on the post-entry market competition. The post-entry equilibrium should be determined by post-entry market structure.
2. Therefore, firm 2 should not be deterred.
3. Firm 1’s commitment of \(k_1 - k_2\) is not reliable. Also firm 2 can make commitment to threaten firm 1. The first-mover advantage does not necessarily belong to incumbent.

8.4.5 Dixit 1980

If firm 2 is not convinced that \(k_2 = 0\), if firm 2 enters, then firm 1 cannot choose \(k_2\) to deter firm 2’s entry. Consider a 2-period model:

\[
t = 1: \quad \text{Firm 1} \text{ chooses } k_1.
\]

\[
t = 2: \quad \text{Cournot competition}: \quad \text{Firms 1 and 2 determine } q_1, q_2 \text{ simultaneously.}
\]

Firm 1’s MC curve in \(t = 2\) is
In both cases $a_i = k$. Therefore, firm 1 does not over-invest (choose $k > a_i$) to threaten firm 2’s entry.

8.4.6 Capital replacement model of Eaton/Lipsey (1986)

It is also possible that an incumbent will replace its capital before the capital is complete depreciated as a commitment to discourage the entry of a potential entrant.

$t = 1, 2, 3, \ldots$.

In each period $t$, if only one firm has capital, the firm earns monopoly profit $H$.

If both firms have capital, each earns duopoly profit $L$.

Each firm can make investment in each period $t$ by paying $F_i$.

Denote by $P_i^L(C_i)$ the profit (cost) of firm $i$ in period $t$.

$$\Pi_t = \sum_{i=0}^{\infty} \rho^i (P_i^L - C_i)$$

$P_i^L = \begin{cases} 0 & \text{no capital} \\ L & \text{duopoly} \\ H & \text{monopoly} \end{cases}$

Assumption 1: An investment can be made for 2 periods with no residual value left

Assumption 2: $2L < F < H$

Assumption 3: $F/H < \rho < \sqrt{(F-L)/(H-L)}$

Firm's strategy is in period $t$ if $a_i \in (NL, INV)$. Assume that $a_{i+1} = INV$. We consider only Markov stationary equilibrium.

If firm 2 does not exist, firm 1 will choose to invest (INV) in $t = 1, 2, 3, \ldots$. Given the threat of firm 2’s possible entry, in a subgame-perfect equilibrium, firm 1 will invest in every period and firm 2 will not invest forever.

The symmetrical SPE strategy is such that an incumbent firm invests and a potential entrant does not:

$$a_i = \begin{cases} \text{INV} & \text{if } a_{i+1} = \text{INV} \\ \text{NL} & \text{otherwise} \end{cases}$$

Proof: that the above strategy is optimal if the opponent plays the same strategy:

$$\Pi_1 = \frac{H - F}{1 - \rho} \Pi_2 = 0$$

If firm 1 deviates and chooses $a_i = \text{NL}$, $r_i$ becomes $H < (H - F)/(1 - \rho)$ (by Assumption 3), because firm 2 will invest and become the monopoly.

If firm 2 deviates and chooses $a_i = \text{INV}$, then $r_2 = (1+\rho)(L - F) + \rho^2 H < 0$ (also by Assumption 3).

8.4.7 Judo economics

The (inverse) market demand is $p = 100 - Q$.

$t = 1$: Firm 2 (entrant) determines whether to enter and if enters, its capacity level $k$ and price $p'$. $t = 2$: Firm 1 (incumbent) determines its price $p$. Assume that firm 1 has unlimited capacity and, if $p = p'$, all consumers will purchase from firm 1.

$$p' = \begin{cases} 100 - p & \text{if } p \leq p' \\ 100 - k - p' & \text{if } p > p' \end{cases}$$

Backward induction:

1. If firm 1 decides to enter, he chooses $p_1'$ and $x_1' = p_1'(100 - k - p_1')$.

2. If firm 1 decides to accommodate firm 2, he chooses to maximize $x_1'' = p_1''(100 - k - p_2')$.

$$\max_{p'} (100 - k - p') \rightarrow \text{FOC: } 0 = 100 - k - 2p_1' \Rightarrow p_1'' = \frac{100 - k}{2} \Rightarrow x_1'' = \frac{(100 - k)^2}{4}$$

Firm 1 will accommodate firm 2 if $x_1'' > x_1'$, or if $\frac{(100 - k)^2}{4} > p_1''(100 - p)$, whence $x'' = x''(p) > 0$.

If firm 2 chooses a small $k$ and a large enough $p'$, firm 1 will accommodate.
At \( t = 1 \), firm 2 will choose \((k_2, p_2)\) such that
\[
\max p' k \text{ subject to } \frac{(100 - k_2)^2}{4} \geq p'(100 - p')
\]

### 8.4.8 Credible Spatial Proximity

Suppose that the incumbent is a monopoly in two markets selling substitute products \( j = 1, 2 \). If an entrant enters one of the two markets, the entrant will gain the market in order to protect the monopoly profit of the other market (product 2).

Reason: If the entrant goes in the market 1, the Bertrand competition will force \( p_2 \) down to its marginal cost. As discussed in the monopoly chapter, the demand of product 2 will be reduced.

#### Example
Suppose that firm 1 has a Chinese restaurant \( C \) and a Japanese restaurant \( J \) in a small town, both are monopolies. There are two consumers (assumed to be price takers), \( a \) and \( b \),
\[
\begin{align*}
I^a &= \left\{ \begin{array}{ll}
\beta - L^C & \text{dine at } C \\
\beta - L^J - p^J & \text{dine at } J
\end{array} \right. \\
I^b &= \left\{ \begin{array}{ll}
\beta - L^C & \text{dine at } C \\
\beta - L^J - p^J & \text{dine at } J
\end{array} \right. \\
\beta > L^C > 0
\end{align*}
\]

\( \beta \) is the utility of dinner and \( L \) is the distance if one goes to a less preferred restaurant.

#### Monopoly Equilibrium
- \( I^C = 0 \) and \( I^J = 0 \), \( p^C = \beta \), \( p^J = \beta - L^J \).

Suppose now that firm 2 opens a Chinese restaurant in the same town.

1. If firm 1 does not close its Chinese restaurant, the equilibrium will be \( I^C = 0 \), \( I^J = 0 \), \( p^C = \beta \), \( p^J = \beta - L^J \).
2. If firm 1 closes its Chinese restaurant, the equilibrium will be \( I^C = 0 \), \( I^J = 0 \), \( p^C = \beta - L^J \), \( p^J = \beta - L^J \).

The conclusion is that firm 1 will close its Chinese restaurant.

### 8.5 Contestable Market of Baumol/Pirenz/Willig (1982)

Deregulation trend in US in later 1970s.

- Airline industry: Each line is an individual industry.

- Contestable market: In certain industries entry does not require any sunk cost. Incumbent firms are constantly faced with threats of hit-and-run entry and hence behave like competitive firms (making normal profits).

#### Assumptions
- Potential entrants and an incumbent produce a homogeneous product and have the same cost function \( T C(q) = F + C q \).
- The market demand is \( p = a - Q \).

An industry configuration: \((p^*, q^*)\)

**Feasibility:**
- \((p^*, q^*) = (p^* - q^*) - (F + C q^*) \geq 0 \)
- \((p^* - q^*) < (F + C q^*) \geq 0 \)

A contestable-market equilibrium: A feasible, sustainable configuration.

#### Extension
- More incumbent firms.
- More than 1 products.

#### Comments
- If there are sunk costs, the conclusions would be reversed. See Stiglitz (1967) discussed before.
- If incumbents can respond to hit-and-run entries, they can still make some positive profits.

### 8.6 A Taxonomy of Business Strategies

- Low, Gama, and Kempster (1985), *“Multimarket Oligopoly: Strategy Substitutes and Complements*" JBE.

A 2-period model: Firm 1 chooses \( k_1 \) at \( t = 1 \) and firms 1 and 2 choose \( x_1, x_2 \) simultaneously at \( t = 2 \).

\[\Pi_t = \Pi_t(1, x_1, x_2), \Pi_2 = \Pi_2(1, x_1, x_2)\]

The reduced payoff functions at \( t = 1 \) are

\[\Pi_1 = \Pi_1(1, x_1, x_2), \Pi_2 = \Pi_2(1, x_1, x_2)\]

Where \( x_1(1) \) and \( x_2(1) \) are the NE at \( t = 2 \), given \( k_1 \).

#### 8.6.1 Deterrence Case

If firm 1 decides to deter firm 2, firm 1 will choose \( k_1 \) to make \( \Pi_1 = \Pi_1(1, x_1(1), x_2(1)) \).

#### 8.6.2 Accommodation Case

If firm 1 decides to accommodate firm 2, firm 1 will choose \( k_1 \) to make \( \Pi_1 = \Pi_1(1, x_1(1), x_2(1)) \).

#### 8.7 Limit Pricing as Cost Signaling

#### 8.7.1 Assumptions of the model

- Each period's demand is \( p = 10 - q \).
- Firm 1 is the incumbent, a monopoly in \( t = 1 \).
- Firm 2 decides whether to enter in \( t = 2 \). If firm 2 enters, the market becomes Cournot competition.

#### 8.7.2 Complete Information Case

If there is no uncertainty, firm 1 will choose the monopoly quantity at \( t = 1 \), i.e.,

\[q_1(1) = 5, q_2(1) = 5, \pi_1(1) = 25, \pi_2(1) = 10, \pi_1(1) = 25 \]

At \( t = 2 \), the duopoly equilibrium for firm 1 with \( q_1 = 0 \) and firm 2 (as \( q_1 = 1) \) is

\[p = 10 - q_2, \pi_1(1) = (10 - 2), \pi_2(1) = 0 \]

The duopoly equilibrium for firm 1 with \( q_1 = 4 \) and firm 2 (as \( q_1 = 1 \)) is

\[p = 10 - q_2, \pi_1(1) = (10 - 2), \pi_2(1) = 0 \]

Therefore, firm 2 will enter if \( q_1 = 4 \) and not enter if \( q_1 = 0 \).

The high cost firm 1 has incentives to confuse firm 2.
CHAPTER 9
RESEARCH AND DEVELOPMENT (R&D)

Production and cost functions are black boxes created by economists. Investigating R&D processes helps us to open the boxes.

Process innovation: An innovation that reduces the production cost of a product.

Product innovation: An innovation that creates a new product.

The distinction is not essential. A process innovation can be treated as the creation of new intermediate products that reduce the production costs. On the other hand, a product innovation can be regarded as an innovation that reduces the production cost of a product from infinity to a finite value.

9.1 Classification of Process Innovations

Consider a Bertrand competition industry. Inverse demand function: \( P = a - Q \).

In the beginning, all firms have the same technology and \( P_0 = C_0 \). Suppose that an inventor innovates a new production procedure so that the marginal production cost reduces to \( c < C_0 \).
2 Cases:

**Drastic Innovation** (large or major innovation): If \( P_w(c) = \frac{a+c}{2} > a - P_h \), then the innovator will become a monopoly.

**Non-draastic Innovation** (small or minor innovation): If \( P_w(c) = \frac{a+c}{2} > a - P_h \), then the innovator cannot charge monopoly price and has to set \( P = a - c \).

A drastic innovation will reduce the market price. A non-draastic innovation will not change the Bertrand equilibrium price. In both cases the innovator makes positive profits.

### 9.2 Innovation Race

**Assumptions:**

1. 2 firms compete to innovate a product.
2. The value of the patent right to the product is \( V \). 4
3. To compete, each firm has to spend \( $I \) to establish a research lab.
4. The probability of firm \( i \) innovating is \( \alpha \). The events of firms being successful is independent.
5. If only one firm succeeds, the firm gains \( $V \). If both succeed, each gains 0.5 \( V \). If a firm fails, it gains 0.
6. The entry is sequential. Firm 1 decides first and then firm 2 makes decision.

#### 9.2.1 Market equilibrium

\[ E\pi_1(n) : \text{Expected profit of firm } k \text{ if there are } n \text{ firms competing.} \]

\[ E\pi_1(1) - \alpha V - I = \begin{cases} I & \text{if } \alpha V \geq I \\ 0 & \text{if } \alpha V < I \end{cases} \]

\[ E\pi_2(2) = \frac{(2-\alpha)}{2} V - I \Rightarrow \pi = \begin{cases} I & \text{if } \frac{(2-\alpha)}{2} V > I \\ 0 & \text{if } \frac{(2-\alpha)}{2} V < I \end{cases} \]

#### 9.2.2 Social Optimal

If there are more firms, the probability of success is higher. On the other hand the investment expenditure will be higher. \( E\pi(n) \): The expected social welfare if there are \( n \) firms attempting.

\[ E\pi(1) - E\pi(n) - \alpha V - I \quad E\pi(2) - 2\alpha(1-\alpha)V + \alpha^2 V - 2I \]

\[ E\pi(2) \geq E\pi(1) \quad \text{if and only if } \alpha(1-\alpha) \geq I \]

(i): Social optimal is 1 firm, the same as market equilibrium number of firms.

(ii): Social optimal is 1 firm, market equilibrium has 2 firms.

(iii): Social optimal is 2 firms, the same as market equilibrium number of firms.

Area (ii) represents the market inefficient area.

#### 9.2.3 Expected date of discovery

Suppose that the R&D race will continue until the discovery.

\[ ET(n) : \text{Expected date of discovery if there are } n \text{ firms.} \]

\[ ET(1) = \alpha + (1-\alpha)\alpha^2 + (1-\alpha)^3\alpha^3 + \cdots = \alpha \sum_{n=1}^{\infty} (1-\alpha)^{n-1} = \frac{\alpha}{1-(1-\alpha)^2} = \frac{1}{\alpha} \]

\[ ET(2) = \alpha(2-\alpha) + (1-\alpha)^2\alpha(2-\alpha)2 + \cdots = \alpha(2-\alpha) \sum_{n=1}^{\infty} (1-\alpha)^{n-1} = \frac{1}{\alpha(2-\alpha)} < ET(1) \]

#### 9.3 Cooperation in R&D

Firms' cooperation in price setting is against anti-trust law. Cooperation in R&D activities usually is not illegal. Therefore, firms might use cooperation in R&D as a substitute for cooperation in price setting.

In this subsection we investigate the effects of firms' cooperation in R&D on social welfare.

A 2-stage duopoly game with R&D

\[ t = 1: \text{Both firms decide R&D level, } x_1 \text{ and } x_2 \text{ simultaneously.} \]

\[ t = 2: \text{Both firms engage in Cournot quantity competition.} \]

Market demand is \( P = 100 - Q \). Firm \( i \)'s R&D cost: \( TC_i(x_i) = x_i^2/2 \). Firm \( i \)'s unit production cost: \( c_i(x_i, x_j) = 50 - x_i - \beta x_j \). \( \beta > 0 \). If \( \beta > 0 \), it represents the spillover effect of R&D; If \( \beta < 0 \), it is the inter-firm effect.

#### 9.3.1 Noncooperative R&D equilibrium

When firms do not cooperate in R&D, they decide the R&D levels, independently of each other. We solve the model backward.

At \( t = 2, x_1 \) and \( x_2 \) are determined. The Cournot equilibrium is such that

\[ \Pi_i(x_i, x_j) = \left( \frac{100 - 2x_i - x_j}{9} \right) - T C_i(x_i) \]

Substituting the cost function, we obtain the reduced profit function of \( t = 1 \):

\[ \Pi_i(x_i, x_j) = \left( \frac{100 - 2(50 - x_i - x_j) + (50 - 2x_i - x_j)}{9} \right) \]

\[ = \left( \frac{50 - 2x_i + (2-\beta)x_j}{9} \right) \]

At \( t = 1 \), firm \( i \) chooses \( x_i \) to maximize \( \Pi_i(x_i, x_j) \). FOC is:

\[ \frac{\partial \Pi_i}{\partial x_i} = 0 \]

\[ \frac{\partial \Pi_i}{\partial x_j} = 0 \]

In a symmetric equilibrium, \( x_1 = x_2 = \frac{50(2-\beta)}{4.5} - \beta \frac{x_j}{9} \)

\[ = 0 \]

\[ \alpha(2-\beta)^2 \]

In a symmetric equilibrium:

\[ x_1 = x_2 = \frac{50(2-\beta)}{4.5} - \beta \frac{x_j}{9} \]

\[ \alpha(2-\beta)^2 \]

\[ \alpha(2-\beta)^2 \]

\[ = 0 \]

\[ = 0 \]

\[ \alpha(2-\beta)^2 \]

\[ = 0 \]

\[ \alpha(2-\beta)^2 \]

#### 9.3.2 Cooperative R&D equilibrium

When firms cooperate in R&D, they choose \( x_1 = x_2 = x \) so that:

\[ \Pi_1 + \Pi_2 = \Pi_c = \left( \frac{50(1+\beta)x}{9} \right) \]

\[ \alpha(2-\beta)^2 \]

\[ \alpha(2-\beta)^2 \]

\[ \alpha(2-\beta)^2 \]
Then they decide the level of \( x \) to maximize \( \Pi(x) \). FOC is

\[
\frac{\partial \Pi}{\partial x} = 0 - \frac{2(1+\beta)(50 + (1+\beta)x)}{9} - x.
\]

Denote by \( x^* \) the optimal level of \( x \).

\[
x_1 = x_2 = x^* = \frac{50(1+\beta)}{4.5 + (1+\beta)^2}, \quad c_2 = c_1 = \frac{50(4.5 - 2(1+\beta)^2)}{4.5 + (1+\beta)^2}.
\]

\[
P^* - c^* = Q^* = \frac{75}{4.5 + (1+\beta)^2}, \quad \Pi_1 = \Pi_2 = \Pi^* = \frac{225(90 - 2(1+\beta)^3)}{4.5 + (1+\beta)^2}.
\]

Conclusions:

1. \( \Pi^* > \Pi^c \).
2. If \( \beta > 0.5 \) then \( x^* > x^c^* \) and \( Q^* > Q^c^* \).
3. If \( \beta < 0.5 \) then \( x^* < x^c^* \) and \( Q^* < Q^c^* \).

When \( \beta > 0.5 \), consumers will be better off to allow R&D cooperation; social welfare will definitely increase. When \( \beta < 0.5 \), consumers will be worse off to allow R&D cooperation; but the social welfare also depends on the change in firms’ profits.

### 9.4.4 Patents

#### 9.4.4.1 Nondisclosure 1969 partial equilibrium model

\[
P = a - Q: \text{Demand function of a Bertrand competition market.}
\]

- \( c \): Unit production cost before R&D.
- \( x \): R&D expenditure.
- \( M(x) \): Total cost after R&D.
- \( D(x) \): Demand function due to monopoly (Bertrand competition).

Assume that the innovation is non-draconic.

\[
P M(x) = x(a-c) \quad \text{and} \quad DL(x) = x^2/2.
\]

**Innovator’s problem:**

\[
\max_t \pi(x; T) = \sum_{t=0}^{T-1} M(x) - TC(x) - \frac{1}{1-\rho} (\alpha - c)x - \frac{1}{1-\rho} x^2/2.
\]

FOC

\[
1 - \frac{1}{1-\rho} (\alpha - c) - \frac{1}{1-\rho} x = 0 \implies x = 1 - \frac{1}{1-\rho} (\alpha - c).
\]

Comparative statics:

\[
\frac{\partial x}{\partial \alpha} > 0, \quad \frac{\partial x}{\partial c} = 0, \quad \frac{\partial x}{\partial M} < 0, \quad \frac{\partial x}{\partial \Pi} > 0.
\]

**Social optimal duration of patents:**

\[
W(T) = \sum_{t=0}^{T} \rho^{t+1} M(x) + \sum_{t=1}^{\infty} \rho^{t+1} DL(x)
\]

\[
\max_{x} x (a-c)x - \frac{1}{1-\rho} x^2/2 \quad \text{subject to} \quad x = 1 - \frac{1}{1-\rho} (\alpha - c).
\]

Eliminating \( x \):

\[
\max_{x} \frac{(a-c)x - 1/\rho - x^2/2}{1-\rho} \quad \text{subject to} \quad x = 1 - \frac{1}{1-\rho} (\alpha - c).
\]

**Comparison between \( \tau_f \) and \( \tau_f^* \):**

\[
9 \times (\tau_f^* - \tau_f) > 0.
\]

Therefore, firms will prefer per-unit fee licensing.

The reason is: In the case of fixed-fee licensing, \( a_1 + c_1 \) and \( P \) are such that firm 1’s total profit is smaller than that of per-unit fee licensing.

### 9.5 Licensing

#### 9.5.1 Equilibrium without licensing

\[
\begin{align*}
\tau_1 &= \frac{a - c + 2c}{3}, \quad \tau_2 = \frac{a - c + 2c}{3}, \quad P = \frac{a + 2c - \phi}{3},
\end{align*}
\]

#### 9.5.2 Equilibrium with per-unit fee licensing

#### 9.5.3 Equilibrium with fixed-fee licensing

#### 9.6.1 Subsidizing new product development

Sometimes governmental subsidies can have very substantial strategic effects.

(Krugman, 1986, Strategic Trade Policy and the New International Economics, Boeing (L A USfirm) and Airbus (II, an EU firm) are considering whether to develop super-large airliners.

Without intervention, the game is:
If EU subsidies 15% to Airbus to produce, the game becomes:

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>Produce</th>
<th>Don’t Produce</th>
</tr>
</thead>
<tbody>
<tr>
<td>Produce</td>
<td>(50, 0)</td>
<td>(0, 50)</td>
</tr>
<tr>
<td>Don’t Produce</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

There are two equilibria: (Produce, Don’t) and (Don’t, Produce).

9.6.2 Subsidizing process innovation

If we regard the R&D levels $x_1$ and $x_2$ in the R&D cooperation model as the amount of R&D sponsored by governments 1 and 2, it becomes a model of government subsidy competition.

9.7 Dynamic Patent Races

Trole section 10.2


9.7.1 Basic model

2 firms compete in R&D to win the patent of a new product. $x_1$: the size of R&D lab established (incurring a continuous cost of $c_1$ per unit of time) by firm 1; $x_2$: the size of the patent per unit of time. $c_1$: constant production cost. $T$: the firm’s discovery time. Assumption: $T_1$ and $T_2$ are independent exponential random variables.

$$T_i \sim \exp(\lambda_i), \text{ density function: } h(x) = \lambda_i \exp(-\lambda_i x),$$

where $h(x)$ is the expected discovery time of firm i.

$$E(T_i) = \frac{1}{\lambda_i}, \quad h(x) > 0, \quad h(x)^2 > 0,$$  

Industry discovery time: $T = \max(T_1, T_2) \sim 1 - \exp(-\lambda_1 x_1 - \lambda_2 x_2)$

$$\text{Prob} \left( T > t \right) = \exp(-\lambda_1 x_1 - \lambda_2 x_2),$$

Prob: $T_i \geq T \iff h(x_1) + h(x_2)$.

Firm 1’s winning probability: $\text{Prob}(T_1 > T) = \frac{h(x_1)}{h(x_1) + h(x_2)}$.

$$\text{Prob}(T_1 \geq T \cap T_2 > T) = \frac{h(x_1)}{h(x_1) + h(x_2)},$$

Using the same derivation as basic model,

$$V_i(x_1, x_2) = \frac{h(x_1) x_1 + h(x_2) x_2 + X(t - T)}{\lambda_i},$$

$$V_i(x_1, x_2) = \frac{h(x_1) x_1 + h(x_2) x_2 + X(t - T)}{\lambda_i}.$$

Comparing the payoff functions reveals that firm 2’s payoff function is essentially the same as that of the basic model. Further comparison between firm 1 and firm 2’s payoff functions reveals that firm 1’s incentives are different in two ways:

Efficiency effect: $\Pi_1(x) - \Pi_1(\lambda_1, \lambda_2) \geq \Pi_1(\lambda_1, \lambda_2)$, firm 1 has more incentives to win the race, and therefore $x_1$ tends to be greater than $x_2$ in this aspect.

Replacement effect: If firm 1 wins, he replaces himself with a new monopoly. Therefore, firm 1 tends to delay the discovery date. $\frac{\partial \Pi_1}{\partial x_1} < 0$ tends to make $x_1$ smaller.

The net effect depends on which one dominates. Following are two extreme cases:

Drastic innovation: $\Pi_1(x, c) = 0$ and $\Pi_2(x, c) = \Pi_2(c)$. No efficiency effect and $x_1 < x_2$.

Almost linear $h(x)$ case: $h(x) = \lambda x, \lambda \rightarrow -\infty$, and $h(x) = h(0)x$.

In this case $h(x)$ and $h(x)^2$ are very large and firm 1 is more concerned with his winning the race rather than replacing itself. Therefore, replacement effect dominates and $x_2 < x_1$. 

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CHAPTER 10
NETWORK EFFECTS, COMPATIBILITY, AND STANDARDS

\[ U^a - U^b(x^*) \Rightarrow U^a - U^b(x^*, x^*, y) \]

Arrow-Debreu general equilibrium model assumes no externalities. With production and/or consumption externalities, we have to modify Arrow-Debreu model.

10.0.3 Concepts
1. Compatibility
   - Standardization
2. Downward-compatibility:
   - Pentium III vs 486
3. Network externalities

10.0.4 A standardization game
Firm A (USA) and firm B (Canada) are choosing a standard for their product.

- Standard
- β-standard
- A \ B
  | α \ β | (α, β) | (β, α) |
  | α   | (α, β) | (β, α) |
  | β   | (β, α) | (α, β) |

1. If \( a, b > \max\{c, d\} \) (battle of the sexes), then \((α, α)\) and \((β, β)\) are both NE. They choose the same standard.

2. If \( c, d > \max\{a, b\} \), then \((α, β)\) and \((β, α)\) are both NE. They choose the different standards.

10.1 Network Externalities
10.1.1 Rohls phone company model

Consumers are distributed uniformly along a line, \( x \in [0, 1] \).

![Graph](image)

Consumers indexed by a low \( x \) are those who have high willingness to pay to subscribe to a phone system.

\[ U^a = \begin{cases} n(1 - x) - p & \text{if } x \text{ subscriber to the phone system} \\ 0 & \text{if } x \text{ does not} \end{cases} \]

where \( n, 0 < n < 1 \): the total number of consumers who actually subscribe,

\( p \): the price of subscribing,

\( x^* \): the marginal consumer who is indifferent between subscribing and not.

In a national expectation equilibrium, \( x = n \) and \( U^a = n(1 - x) - p = 0 \), \( \Rightarrow \) Inverse demand function: \( p = \frac{1}{n} \).

Demand function: \( p = \frac{1}{n} \).

Equilibrium: \( (x_A, x_B) \)

![Graph](image)

- Stable equilibrium: A distribution such that \( x_A + x_B < 1 \).
- Unstable equilibrium: A distribution such that \( x_A + x_B > 1 \).

10.1.2 The standardization-variety tradeoff
Consumers are distributed uniformly along a line, \( x \in [0, 1] \).

<table>
<thead>
<tr>
<th>prefer A</th>
<th>prefer B</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>β</td>
</tr>
</tbody>
</table>

2 brands/standards,

- A: 0 consumers prefer A-standard,
- B: 1 - \( n \) consumers prefer B-standard.

- \( x_A \): number of consumers using A-standard.
- \( x_B \): number of consumers using B-standard.

Δ: the disutility of using a less preferred standard.

\( U^a = \begin{cases} x_A & \text{use A-standard} \\ x_B & \text{use B-standard} \end{cases} \)

Consumer distribution: \( (x_A, x_B) \) such that \( x_A + x_B = 1 \).

B-standard distribution: A distribution such that \( (x_A, x_B) = (0, 1) \).

Incompatible A-B standards distribution: A distribution with \( x_A + x_B > 0 \).

Equilibrium: \( (x_A, x_B) \) such that none wants to switch to a different brand.

- Proposition 10.3: If \( x < 1 \), then both A-standard and B-standard are equilibrium. If \( x > 1 \), then both A-standard and B-standard are not equilibrium.

Proof: If \( x > 1 \) and every one chooses the same brand (either A or B), then none wants to switch to a different brand. If \( x < 1 \), then the cost of switching to a preferred brand is less than the benefit and therefore a standard equilibrium cannot exist.

- Proposition 10.4: If \( x < 1 \), then \( (x_A, x_B) = (a, b) \) is an equilibrium.

Proof: Given the distribution \( (x_A, x_B) = (a, b) \), the utility levels are

\[ U^a = \begin{cases} a - 1 - a - \delta & \text{use A br.} \\ 0 & \text{use B br.} \end{cases} \]

\[ U^b = \begin{cases} a - 1 - b - \delta & \text{use A br.} \\ 0 & \text{use B br.} \end{cases} \]

Therefore, none will switch to a different brand.
Social welfare: \( W(x, a) = aU(x) + B(x) \), where \( U(x) \) is the utility level of A-preferred consumers (B-preferred consumers):

\[
W(A) = W(1, 0) = a + b(1 - \delta) = 1 - \delta b, \\
W(B) = W(0, 1) = a + b(1 - \delta) = a - b\delta, \\
W(AB) = W(1, 1) = a + b(1 - \delta)^2 + b^2 = 1 - b(1 - 2\delta) = 1 - 2\delta b.
\]

Proposition 10.5: If \( a > b \), then \( W(A) > W(B) \).

Proof: If \( W(A) > W(B) \) then \( \delta < 1 \), then \( W(AB) > \max\{W(A), W(B)\} \).

Proposition 10.6: 1. If \( \delta < 1 \), then \( W(AB) > \max\{W(A), W(B)\} \).

Proposition 10.7: If \( \delta < 1 \), then \( W(AB) > \max\{W(A), W(B)\} \).

Remark: When \( a > b \) and \( \delta < 1 \), the social optimum is A-standard. However, both the incompatibility standards and B-standard can also be equilibrium.

10.2.2 Partial compatibility


A PC industry with 2 brands, A and B, and prices \( P_A \) and \( P_B \). Consumers are distributed uniformly along a line, \( \delta \in [0, 1] \).

\[
\delta \text{ prefer } A \quad \text{ prefer } B 
\]

Let \( N_A \) and \( N_B \) be the numbers of software pieces available to computers A and B, respectively.

The utility of consumer \( \delta \) is

\[
U(\delta) = \left\{ \begin{array}{ll}
(1 - \delta)\sqrt{N_A} & \text{if } \delta \text{ buys A-system} \\
(1 - \delta)\sqrt{N_B} & \text{if } \delta \text{ buys B-system.}
\end{array} \right.
\]

In the above, \( \sqrt{N_A} \) can easily be generalized to \( N_A^a \).

Marginal consumer \( \delta \):

\[
U'(A) = (1 - \delta)\frac{\delta}{\sqrt{N_A}} - U'(B) = (1 - \delta)\frac{\delta}{\sqrt{N_B}} \Rightarrow \delta = \frac{\sqrt{N_A}}{\sqrt{N_A} + \sqrt{N_B}}.
\]

Market share: \( \delta_A = \delta \) and \( \delta_B = 1 - \delta \).

If \( N_A \) increases (or \( N_B \) decreases), \( \delta \) will decrease, A’s market share will increase and B’s market share will decrease.

\[
\delta_A = \delta - \frac{\sqrt{N_A}}{\sqrt{N_A} + \sqrt{N_B}}, \quad \delta_B = 1 - \delta - \frac{\sqrt{N_B}}{\sqrt{N_A} + \sqrt{N_B}}.
\]

In this model, there are two monopoly competition software industries, A-software and B-software. Assume that each consumer has \( Y \) dollars to spend on a computer system. If consumer chooses A-system, he has \( E_A = Y - P_A \) to spend on software.

There is a variety effect in each software industry and the number of software piece is proportional to the aggregate expenditure spent on them:

\[
N_A = k_1E_A = k_1(Y - P_A), \quad N_B = k_2E_B = k_1(1 - \delta)(Y - P_B).
\]

Therefore, the equilibrium market shares are

\[
\delta_A = \frac{E_A}{E_A + E_B} = \frac{Y - P_A}{2Y - P_A - P_B}, \quad \delta_B = 1 - \delta = \frac{E_B}{E_A + E_B} = \frac{Y - P_B}{2Y - P_A - P_B}.
\]

Network effect: When \( \delta \) goes down, \( \delta_A \) goes down (\( \delta_B \) goes up), which will turn to reduce \( N_A \) (increase \( N_B \)). Finally, A-users’ utility levels will decrease (B-users’ utility levels will increase).

The network effect here is the same as the variety effects in Dixit/Stiglitz monopolistic competition model.

Duopoly price competition:

The profit functions of firms A and B are

\[
\Pi_A(P_A, P_B) = P_A - \frac{P_A^2}{2Y - P_A - P_B}, \quad \Pi_B(P_A, P_B) = P_B - \frac{P_B^2}{2Y - P_A - P_B}.
\]

The price competition equilibrium is derived in Chou/Shy (1990).

10.3 The Components Model


A51 2 firms, A and B, producing \( X_A \), \( Y_A \), \( X_B \), \( Y_B \).

A52 Marginal costs are 0.

A53 \( X \) and \( Y \) are completely complementary.

A54 3 consumers: AA, AB, BB. You need an \( X \) and a \( Y \) to form a system. 2 situations:

1. Incompatibility: A and B’s products are not compatible. You have to buy \( X_AY_A \) or \( X_BY_B \).

2. Compatibility: A and B’s products are compatible. There are 4 possible systems: \( X_AY_A, X_BY_B, X_BY_A, X_AY_B \).

Consumer \( i,j \)’s utility, \( U_{ij} = AA, AB, BB \), is
10.3.1 Incompatibility

There are only 2 systems: A-system (X,Y), B-system (X,Y).

Equilibrium: (P^*, d^*, d^*), such that
1. P^* maximizes \( \Pi = P^* d^* \).
2. (d^*, d^*) are the aggregate demand of consumers at price \( P^* \).

Lemma 10.1. In an equilibrium, consumer AA (consumer BB) purchases A-system (B-system).

Proof: If in an equilibrium consumer AA purchases B-system, it must be \( P^* < 0 \). In that case, firm A can set \( P^* = 3 \) to attract consumer AA.

Proposition 10.13. There are 3 different equilibria:
1. \( (P^*, 0, 0, 0, d^*, d^*) = (3, 2, 1, 1) \). AA and BB purchase A-system and BB purchases B-system. \( \Pi = 3 \), CS = 3, social welfare is 3.
2. \( (P^*, 0, 0, 0, d^*, d^*) = (2, 3, 1, 1) \). AA purchases A-system and BB purchases B-system. \( \Pi = 2 \), CS = 2, social welfare is 2.
3. \( (P^*, 0, 0, 0, d^*, d^*) = (2, 3, 1, 1) \). AA purchases A-system and BB purchases B-system. \( \Pi = 2 \), CS = 2, social welfare is 2.

10.3.2 Compatibility

4 systems: AA-system (X,Y), BA-system (Y,X), BB-system (X,Y), BB-system (Y,X).

Equilibrium: (P^*, P^*, P^*, P^*, P^*, d^*, d^*, d^*, d^*) such that
1. (P^*, P^*) maximizes \( \Pi = P^* d^* \).
2. (d^*, d^*, d^*) are the aggregate demand of consumers at price \( P^* \).

Proposition 10.14. There exists an equilibrium such that \( P^* = 0 \), \( d^* = d^* \), \( d^* = 0 \), \( \Pi = 2 \), CS = 2, social welfare is 2.

10.3.3 Comparison

1. Consumers are worse off under compatibility.
2. Firms are better off under compatibility.
3. Social welfare is higher under compatibility.

Exercise: a 4-stage game. If at \( t = 1 \) firms determine whether to design compatible components and at \( t = 2 \) they engage in price competition, then they will choose compatibility.

## CHAPTER 11
### ADVERTISING

Advertising is defined as a form of providing information about price, quality, and location of goods and services. 2% of GDP in developed countries, 5% in developing countries, vegetables, etc., < 2% of sales. Cosmetics, detergent, etc., 20-50% of sales. In 1990, GM spent $83 per car, Ford $130 per car, Chrysler $113 per car.

What determines advertising in different industries or different firms of the same industry? Economy of scale, advertising elasticity of demand, etc.


1. Wrong information about product differentiation - increases cost.
2. An entry-deterring mechanism reduces competition.

Tolcher (1964), "Advertising and Competition," JPE
Nelson (1970), "Information and Consumer Behavior," JPE
Nelson (1972), "Advertising as Information," JPE

Nelson:
Search goods: Quality can be identified when purchasing.
Experience goods: Quality cannot be identified until consuming.
Persuasive advertising: Intends to enhance consumer tastes, e.g., diamond.
Informative advertising: Provides basic information about the product.

11.1 Persuasive Advertising

\[ Q(P, A) = \beta A^p P^r, \quad \beta > 0, \quad 0 < r < 1, \quad s < -1. \]

A: expenditure on advertising.
\( c_A \): Advertising elasticity of demand.
\( p_A \): Price elasticity of demand.
\( c \): Unit production cost.

FOC with respect to \( P \):

\[ \frac{\partial \Pi}{\partial P} = -\beta A^p P^r - c_A = 0, \quad \Rightarrow P = \frac{c_A}{r + 1}, \quad \frac{A}{P} = s. \]

FOC with respect to \( A \):

\[ \frac{\partial \Pi}{\partial A} = -c_A \beta A^{p-1} P^r - c_A = 0, \quad A = \frac{c_A}{\beta P^r}, \quad \frac{A}{P} = s. \]

Proposition: The proportion of advertising expenditure to total sales is equal to the ratio of advertising elasticity to price elasticity.

11.1.1 Example: \( b = 0.64, c = 0.5, s = -2, \quad c = 1 \)

\[ Q = 64\sqrt{A} P^{0.5}, \quad P = 0.84 \sqrt{Q}, \quad s = 2, \quad Q^e = 16\sqrt{A}, \quad A^e = 64, \quad \Pi = 16\sqrt{A}, \Rightarrow \]

\[ \Rightarrow 4 = \alpha = \beta = 2, \quad A = 32, \quad A = 32. \]

Social welfare: \( W(A) = 2A + \frac{1}{2}A^2 - A \), Social optimal: \( W(A) = 0, \quad A = 2, \quad W(A) = 0, \quad A = 32. \]

Remark 1. Does CS(A) represent consumers welfare? If it is informative advertising, consumers utility may increase when \( A \) increases. However, if consumers are just persuaded to make unnecessary purchases, the demand curve does not really reflect consumers utility.

2. Crowding-out effect: Consumption for other goods will decrease.
3. A can be interpreted as other utility enhancing factors.

11.2 Informative Advertising

Consumers often rely on information for their purchases. The problem is whether there is too little or too much informative advertising.

Informative advertising level under monopolistic competition equilibrium is social optimal.

In a circular market, informative advertising level is too excessive.

In the case of 2 differentiated products, the result is uncertain.
11.2.1 A simple model of informative advertising

1 consumer wants to buy 1 unit of a product.
\( p \) the price. \( m \) the value.

\[ U = \begin{cases} \frac{m-p}{0} \\
\end{cases} \]

If the consumer does not receive any advertisement, he will not purchase.
If he receives an advertisement from a firm, he will purchase from the firm.
If he receives 2 advertisements from 2 firms, he will randomly choose one to buy.

2 firms, with production cost is 0, informative advertising cost is \( A \).
Each chooses either to advertise the product or not to advertise it.

\[ n_i = \begin{cases} \frac{p-A}{2} & \text{if only firm } i \text{'s ad is received.} \\
\frac{p-A}{2} & \text{if both firms' ad are received.} \\
-A & \text{if firm } i \text{'s ad is not received.} \\
0 & \text{if firm } i \text{'s choice not to advertise.} \\
\end{cases} \]

Let \( \delta \) be the probability that an advertisement is received by the consumer.

\[ E_r = \begin{cases} 0 & \text{if only firm } i \text{'s choice to advertise.} \\
\frac{1}{2} & \text{if both firms choice to advertise.} \\
0 & \text{if firm } i \text{'s choice not to advertise.} \\
\end{cases} \]

If \( p/A > 1/4 \)  \( \Rightarrow n(2) > 0 \)  \( \Rightarrow \) at least one firm will choose to advertise.
If \( p/A > 2/\delta(2-\delta) \)  \( \Rightarrow r(2) > 0 \)  \( \Rightarrow \) both firms will choose to advertise.

\[ \frac{2}{\delta(2-\delta)} \] 2 firms in equilibrium.

\[ (1,0,0,0,0) \]

Welfare comparison:

\[ EW = \begin{cases} \frac{\delta - 2}{\delta} m - 2A = W(2) & \text{if 2 firms advertise.} \\
\delta m - A = W(1) & \text{if only one firm advertises.} \\
0 & \text{if no firm advertises.} \\
\end{cases} \]

If \( m/A > 1/\delta \)  \( \Rightarrow W(1) > 0 \), \( \Rightarrow \) social optimal is at least one firm advertises.
If \( m/A > 1/\delta(2-\delta) \)  \( \Rightarrow W(2) > W(1) \), \( \Rightarrow \) social optimal is both firms advertise.

11.3 Targeted Advertising

11.3.1 The model

2 firms, \( i = 1,2 \), producing differentiated products.
2 groups of consumers: \( E \) experienced consumers and \( N \) inexperienced consumers.
\( \theta E \) of experienced consumers are brand 1 oriented, \( \theta < 1 \).
\( 1-\theta E \) of experienced consumers are brand 2 oriented.

2 advertising methods: \( P \) (persuasive) and \( I \) (informatique).
Each firm can choose only one method.

A1: Persuasive advertising attracts only inexperienced consumers. If only firm 1 chooses \( P \), then all \( N \) inexperienced consumers will purchase brand 1. If both firms 1 and 2 choose \( P \), all will have \( N/2 \) inexperienced consumers.

A2: Informatique advertising only the experienced consumers who are oriented toward the advertised brand, i.e., if firm 1 (firm 2) chooses \( 1, \theta E \) (\( 1-\theta E \)) experienced consumers will purchase brand 1 (brand 2).

ASS: A firm earns \$1 from each customer.

From the assumptions we derive the following duopoly advertising game:

\[ \begin{bmatrix}
\text{firm 1} & \text{firm 2} \\
\frac{P}{I} & \begin{bmatrix}
\begin{pmatrix}
N/2, 0/2 \\
\theta E, (1-\theta)E \\
\end{pmatrix}
\end{bmatrix}
\end{bmatrix} \]

11.3.2 Proposition 11.5

1. \( (P, P) \) is a NE if only if \( N/2 \geq \theta E \) and \( N/2 \geq (1-\theta)E \) or \( 1 - N/2E \leq \theta \leq \frac{N}{2E} \).

2. \( (I, I) \) is a NE if only if \( \theta \leq \frac{N}{4E} \) and \( \theta \leq (1-\theta)E \) or \( \theta \leq \frac{N}{2E} \).

3. \( (P, I) \) is a NE if only if \( N/2 \geq (1-\theta)E \) and \( N \geq \theta E \) or \( \theta \leq \min\{1 - N/2E, N/4E\} \).

4. \( (I, P) \) is a NE if only if \( N/2 \leq \theta E \) and \( N \geq (1-\theta)E \) or \( \theta \geq \max\{1 - N/2E, N/4E\} \).

11.4 Comparison Advertising

Comparison advertising: The advertised brand and its characteristics are compared with those of the competing brand.
It became popular in the printed media and broadcast media in the early 1970s.
Advantages of comparison advertising:
1. Provides consumers with lower-cost means of evaluating available products.
2. Increases purchase intentions and lowers the perceived risk of choosing new brands.
3. Forces the manufacturers to build into the producible attributes consumers want.

11.4.1 Application of the targeted ad model to comparison ad

Plain ad: \( \rightarrow \) Persuasive ad, aiming at inexperienced consumers.
Comparison ad: \( \rightarrow \) Targeted ad, aiming at experienced consumers.

Applying Proposition 11.5 and the diagram, we get the following results:
1. Both firms will use comparison ad only if \( E > 2N \).
2. If \( 2E < N \), both firms will use plain ad.
3. Comparison ad is used by the popular firm and plain ad is used by the less popular firm in other cases in general.

11.5 Other Issues

11.5.1 Can information be transmitted via advertising?

Search goods: False advertising is unlikely.
Experience goods: Producers will develop persuasive methods to get consumers to try their products.

Facts: 1. Due to asymmetry of information about quality, consumers cannot directly rely on ads.
2. High-experience products buyers are mostly experienced consumers.

Low-quality brands are more frequently purchased and firms producing low-quality brands advertise more intensively. \( \Rightarrow \) There is a negative correlation between advertising and the quality of advertised products.
CHAPTER 13

QUALITY

Quality is a vertical differentiation character.

2 period game:
At \( t = 1 \), firms A and B choose the quality levels, \( 0 \leq a < b \leq 1 \), for their products. At \( t = 2 \), they engage in price competition.

Consumers in a market are distributed uniformly along a line of unit length.

Each point \( x \in [0, 1] \) represents a consumer.

\[
U_x = \begin{cases} 
    ax - P_A & \text{if } x \text{ buys from A,} \\
    bx - P_B & \text{if } x \text{ buys from B.}
\end{cases}
\]

The marginal consumer \( x \) is indifferent between buying from A and from B. The location of \( x \) is determined by

\[
ax - P_A - bx - P_B = 0 \Rightarrow x = \frac{P_A - P_B}{a - b}. \tag{7}
\]

If the location of \( x \) divides the market into two parts: \([0,\hat{x}]\) is firm A’s market share and \((\hat{x},1]\) is firm B’s market share.

\[
\begin{array}{c|c|c}
\text{A’s share} & \text{B’s share} \\
0 & \hat{x} & 1
\end{array}
\]

Assume that the marginal costs are zero. The payoff functions are

\[
\Pi_A(P_A, P_B, a, b) = P_A x, \quad \Pi_B(P_A, P_B, a, b) = P_B (1 - x).
\]

The FOCs are (the SOC’s are satisfied)

\[
\frac{\partial \Pi_A}{\partial P_A} = P_B - \frac{2P_B}{a - b} = 0, \quad \frac{\partial \Pi_B}{\partial P_B} = 1 - \frac{2P_B}{a - b} = 0.
\]

The equilibrium is given by

\[
P_A = \frac{b - a}{3}, \quad P_B = \frac{2(b - a)}{3}, \quad \Rightarrow \hat{x} = \frac{1}{3}.
\]

The reduced profit functions at \( t = 1 \) are

\[
\Pi_A = \frac{(b - a)}{3}, \quad \Pi_B = \frac{4(b - a)}{9}.
\]

Both profit functions increase with \( b - a \). Moving away from each other will increase both firms’s profits. The two firms will end up with maximum profit differentiation \( a = 0 \) and \( b = 1 \) in equilibrium.

Modifications: 1. High quality products are associated with high unit production cost.
2. Consumer distribution is not uniform.

12.2 Quality-Signalling Games

There are one unit of identical consumers each with utility function

\[
U = \begin{cases} 
    H - P & \text{if } x, \\
    L - P & \text{if not } x.
\end{cases}
\]

\( C_H > C_L \geq 0 \): Unit production costs of producing high- and low-quality product.

AS1: The monopolist is a high-quality producer.

AS2: \( H > L > C_H \).

Signalling equilibrium: \( P^m = H \) and \( Q^m = \frac{H - C_L}{1} \).

Proof: 1. For a low-quality monopolist, to imitate the high-quality monopolist is not worthwhile:

\[
\Pi_H(P^m, Q^m) = (P^m - C_H)Q^m = \frac{H - C_H}{1} - L - C_L - \Pi_H(L, 1).
\]

2. The high-quality monopolist has no incentives to imitate a low-quality monopolist:

\[
\Pi_H(P^m, Q^m) = (P^m - C_H)Q^m = \frac{H - C_H}{1} - L - C_L - \Pi_H(L, 1).
\]

The last inequality is obtained by cross multiplying.

The high-quality monopolist has to reduce its quantity to convince consumers that the quality is high.

If the information is perfect, he does not have to reduce quantity. The quantity reduction is needed for signalling purpose.

12.3 Warranties


Higher-quality firms offer a larger warranty than do low-quality firms.


A comprehensive analysis of a monopoly that can offer a warranty for its product.
13 Pricing Tactics

13.1 Two Part-Tariff

Og (1971), “A Disneyland Dilemma: Two Part Tariff for a Mickey Mouse”, QJE.

In addition to the per unit price, a monopoly firm (amusement parks, sports clubs can set a second pricing instrument (membership dues) in order to be able to extract more consumer surplus.

\[ P \text{ price, } d \text{ membership dues, } m \text{ consumption of other goods.} \]

Budget constraint: \( m + d + \frac{Q}{P} = 1 \)

Utility function: \( U = m + 2\sqrt{Q} \)

\[ \max Q = 1 - \phi \frac{P^*}{Q} + 2\sqrt{Q} \Rightarrow \text{demand function: } P = \frac{1}{Q^2} \]

13.1.1 No club annual membership dues

Club capacity: \( K \)

\[ \text{Club profit: } \Pi(Q) = PQ - \sqrt{K} \]

\[ \max Q \Pi(Q) \Rightarrow Q_m = K, P_m = \frac{1}{\sqrt{K}}, \Pi_m = \sqrt{K} \]

13.1.2 Annual membership dues

\[ \max Q_m(\phi) = \phi \text{ subject to } 1 - \phi + 2\sqrt{K} > I = I_0 \]

\[ \Rightarrow \phi^* = 2\sqrt{K} = I_0 \]

Using annual membership dues, the monopoly extracts all the consumer surplus, like the first degree price discrimination.

13.1.3 Two-part tariff

There are two problems with membership dues:
1. It is difficult to estimate consumers’ utility function and the profit maximizing \( \phi^* \).
2. Consumers are heterogeneous.

Therefore, the monopoly offers a “package” of \( Q_3 < K \) and annual fee \( q_3 < \phi^* \).

In addition, the monopoly offers an option to purchase additional quantity for a price \( P_4 \).

13.2 Peak-Load Pricing

High- and Low-Seasonal Demand Structure: \( P^H = A^H - Q^H, P^L = A^L - Q^L \), \( A^H > A^L \).

Cost Structure: \( TC(Q^H, Q^L, K) = c(Q^H + Q^L) + rK \) for \( 0 < Q^H, Q^L < K \).

\[ \text{unit variable cost, } c; \text{ capacity, } r; \text{ unit capacity cost.} \]

\[ \max \Pi^H(Q^H) = P^H \text{ and } \Pi^L(Q^L) = \text{subject to } 0 < Q^H, Q^L < K. \]

\[ \text{FOC: } \frac{\partial \Pi^H(Q^H)}{\partial Q^H} = c + r, \frac{\partial \Pi^L(Q^L)}{\partial Q^L} = c, Q^L < Q^H \]

\[ \Rightarrow P^H = A^H + c + r \frac{2}{2} \]

Regulation for efficiency: \( P^H = c + r \frac{2}{2} \)

\[ n \text{-period case: } \Pi^H(Q^H) = 1 + \frac{2}{n} \]

\[ \Pi^L(Q^L) = c \]

Modification: Substitutability between high- and low-seasonal demand.

CHAPTER 14

MARKETING TACTICS : BUNDLING, UPGRADING, AND DEALERSHIPS

14.1 Bundling and Tying

Bundling: Firms offer for sale packages containing more than one unit of the product. It is a form of nonlinear pricing (2nd degree price discrimination).

Tying: Firms offer for sale packages containing at least two different (usually complementary) products.

Examples: Car and car radio, PC and software, Book and T-shirt.

14.1.1 How can bundling be profitable?

Monopoly demand: \( Q(P) = 4 - P \), \( MC = 0, \)

Monopoly profit maximization: \( P^m = 2 - Q^m, \Pi^m = 4. \)

Bundling 4-unit package for \( SS \):
(1) The consumer will have no choice but buying the package.
(2) The monopoly profit becomes \( II > II^* \).

The monopoly in this case uses bundling tactics to extract all the consumer surplus.

14.1.2 How can tying be profitable?

A monopoly sells goods \( X \) and \( Y \).

2 consumers, \( i = 1, 2 \), who have different valuations of \( X \) and \( Y \).
Valuations: \( V_2 = H, V_2^2 = L, V_2^3 = L, V_2^4 = H, V_2 > L > 0 \).

Assume that consumers do not trade with each other.

Equilibrium without tying:

\[ P^m = P^m \text{ subject to } H \text{ if } H > 2L \text{ and } \Pi^m = \left\{ \begin{array}{ll}
4L & \text{if } H > 2L,
\end{array} \right. \]

Equilibrium with tying: \( P_f = P_{b,ty}, \Pi_f = Q_{b,ty} \)

\[ P_{b,ty} = H + L, \text{ and } \Pi^m = \left\{ (H + L) > II^* \right\}. \]

14.1.3 Mixed tying

Assume that consumers do not trade with each other.

**Equilibrium without tying:**

1. \( P_1 - P_2 = 3, Q_1 - Q_2 = 2, \Pi(1) = -12 \).
2. \( P_2 = P_3 = 4, Q_2 = Q_3 = 1, \Pi(2) = 8 < \Pi(1) \).
Therefore, \( P^{eq}_1 = 5, P^{eq}_2 = 6, Q^{eq}_1 = Q^{eq}_2 = 1, \Pi^{eq} = 14 > \Pi^{eq}_1 = 12 \).

**Equilibrium with pure tying, \( P_1 = P_{t1}, Q_1 = Q_{t1} \):**

1. \( P_2 = 6, Q_2 = 1, \Pi(2) = 6 < \Pi(1) \).
Therefore, \( P_2 = 4, Q_2 = 3, \Pi_2 = 12 \).

**Equilibrium with mixed tying:**

1. \( P^{eq}_1 = 4, P^{eq}_2 = 6, Q^{eq}_1 = Q^{eq}_2 = 1, \Pi^{eq} = 14 > \Pi^{eq}_1 = 12 \).

But mixed tying is not always as profitable as pure tying.

### 14.1.4 Tying and foreclosure (\( F(t) \))

US antitrust laws prohibit bundling or tying behavior whenever it leads to a reduced competition. What is the connection between tying and reduced competition?

2 computer firms, X and Y, and a monitor firm Z (compatible with X and Y),

2 consumers \( i = 1, 2 \) with utility functions

\[
U_i^1 = \begin{cases} 3 - P_2 - P_3 & \text{buys X and Z} \\ 1 - P_1 - P_2 & \text{buys Y and Z} \\ 0 & \text{buys nothing} \end{cases} \quad U_i^2 = \begin{cases} 1 - P_2 - P_3 & \text{buys X and Z} \\ 3 - P_2 - P_3 & \text{buys Y and Z} \\ 0 & \text{otherwise} \end{cases}
\]

Bermudan equilibrium with 3 independent firms:

1. \( P_1 = P_2 = 2, P_1 = 1, Q_2 = Q_3 = 1, Q_1 = 2, \Pi_1 = \Pi_2 = \Pi_3 = -2 \).
2. \( P_{t1} = P_{t2} = 2, Q_{t1} = Q_{t2} = 1, Q_{t3} = 2, \Pi_{t1} = \Pi_{t2} = \Pi_{t3} = 0 \).

Assume that firm X buys firm Z and sells X and Z tied in a single package.

Total foreclosure equilibrium:

\( P_{f1}^\infty = 3 - \epsilon, Q_{f1}^\infty = 2, P_{f2}^\infty = \epsilon, Q_{f2}^\infty = 1, \Pi_{f1}^\infty = 2(3 - \epsilon), \Pi_{f2}^\infty = \epsilon \).

Consumer 2 buys one X\&Z and one Y and discards X.

### 14.1.5 Tying and International markets segmentation

Governments trade restrictions like tariffs, quotas, etc., help firms to engage in price discrimination across international boundaries.

A two country, \( k = 1, 2 \), with one consumer in each country.

A world-monopoly producer sells X.

It can sell directly to the consumer in each country or open a dealership in each country selling the product tied with service to the consumer.

The utility of the consumer in each country (also denoted by \( k = 1, 2 \)) is

\[
U_i^k = \begin{cases} B_i + \sigma & \text{if 1 buys X & service} \\ B_i - P^{eq}_i & \text{if 1 buys X only} \\ 0 & \text{if 1 does not buy} \end{cases} \quad \Pi_i^k = \begin{cases} B_i + \sigma - P^{eq}_i & \text{if 2 buys X & service} \\ B_i - P^{eq}_i & \text{if 2 buys X only} \\ 0 & \text{if 2 does not buy} \end{cases}
\]

where \( P^{eq}_i \) are the price with service (without service) in country \( k, k = 1, 2 \), and \( \sigma > 0 \) is the additional value due to service.

AS1 \( B_1 > B_2 \)

AS2 Marginal production cost is 0.

AS3 Unit cost of service provided by the dealership is \( w > 0 \).

No attempts to segment the market:

\[
p^{eq} = \begin{cases} B_1 & \text{if } B_1 < 2B_2 \\ B_2 & \text{if } B_1 > 2B_2 \end{cases} \quad \Pi^{eq} = \begin{cases} 2B_2 - \frac{B_1}{2} & \text{if } B_1 < 2B_2 \\ B_1 & \text{if } B_1 > 2B_2 \end{cases}
\]

Segmenting the market:

\( P^{eq}_1 = B_1 + \sigma, \quad \Pi^* = B_1 + B_2 + 2(\sigma - w) \).

Nonarbitrage condition: \( B_1 - B_2 < 0 \).